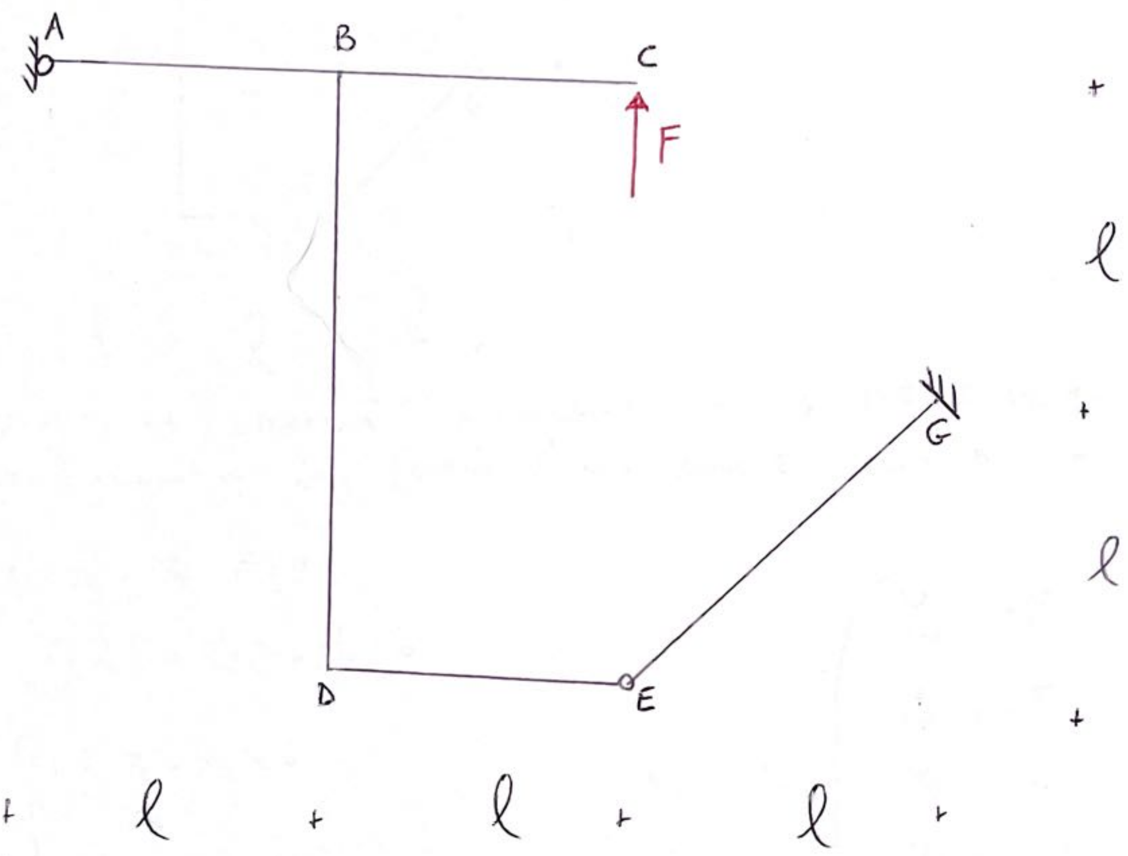
ESERCITAZIONE n°5



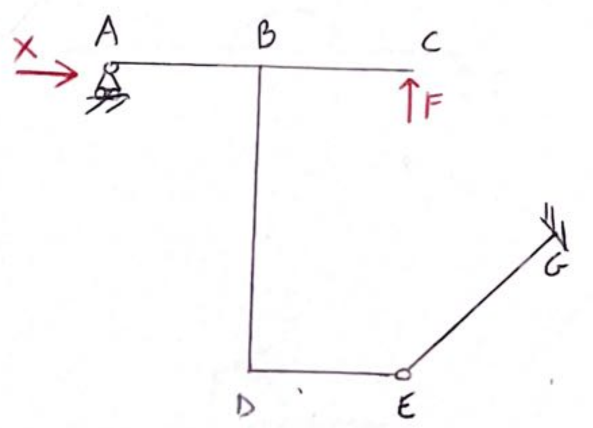
SISTEMA EFFETTIVO

PER POTER APPLICARE IL METODO DELLE FORZE BISOGNA CLASSIFICARE LA STRUTTURA; ABBIAMO 2 CERNIERE E 1 INCASTRO QUINDI $m = 7$ AVENDO 2 CORPI RIGIDI $n = 6$. OSSERVANDO CHE $p = n$ SI HA:

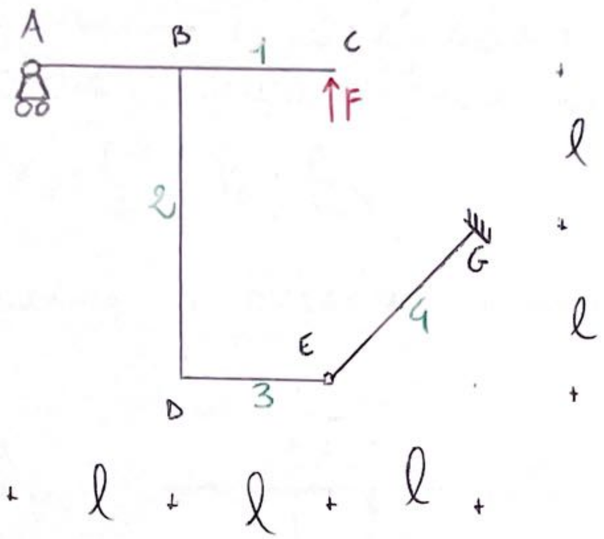
$i = m - p = 1$

IL SISTEMA È 1 VOLTA IPERSTATICO.

SOSTITUIREMO LA CERNIERA IN A CON UN CARRELLO E UNA FORZA INCOGNITA IN DIREZIONE DEL VINCOLO RIMOSSO. STUDIEREMO COSÌ UN SISTEMA ISOSTATICO.



SISTEMA ISOSTATICO EQUIVALENTE



SISTEMA "0"

STUDIO LA STRUTTURA DIVIDENDOLA IN 4 PARTI E IMPONENDO L'EQUILIBRIO SU OGNUNO DI ESSI (LUNGO X NON AVVIENE NULLA DI INTERESSANTE)

$$1\begin{cases}(\uparrow)\ Y_A + Y_B + F = 0\\ (\overset{\curvearrowleft}{A})\ Y_B l + F2l + M_B = 0\end{cases}$$

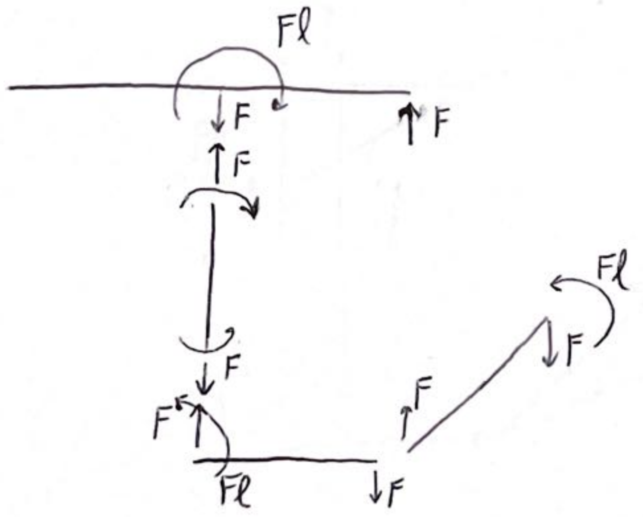
$$2\begin{cases}(\uparrow)\ Y_D - Y_B = 0\\ (\overset{\curvearrowleft}{D})\ M_B + M_D = 0\end{cases}$$

$$3\begin{cases}(\uparrow)\ Y_D - Y_E = 0\\ (\overset{\curvearrowleft}{D})\ -Y_E l + M_D = 0\end{cases}$$

$$4\begin{cases}(\uparrow)\ Y_E + Y_G' = 0\\ (\overset{\curvearrowleft}{G})\ -Y_E l + M_G = 0\end{cases}$$

$$\begin{cases}Y_B = -F\\ Y_D = -F\\ Y_A = 0\\ Y_E = F\\ Y_G' = F\\ M_D = Fl\\ M_B = -Fl\\ M_G = Fl\end{cases}$$

IL DIAGRAMMA DI STRUTTURA LIBERA:

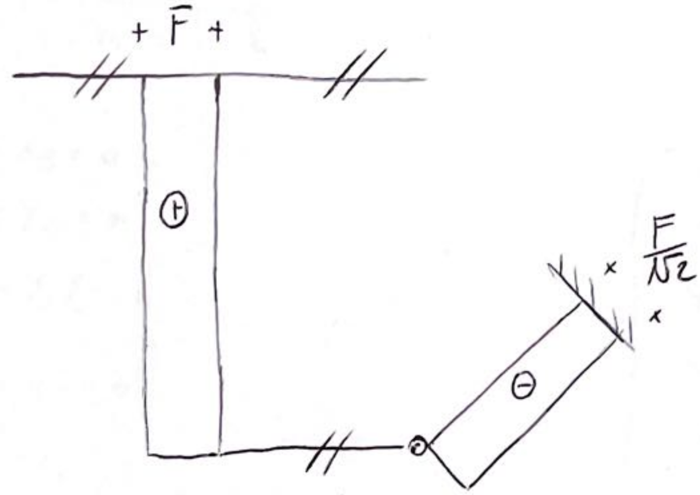


SUL VINCOLO IN G E' POSSIBILE PROIETTARE LA REAZIONE SULL'ASSE DELLA TRAVE, AVREMO:

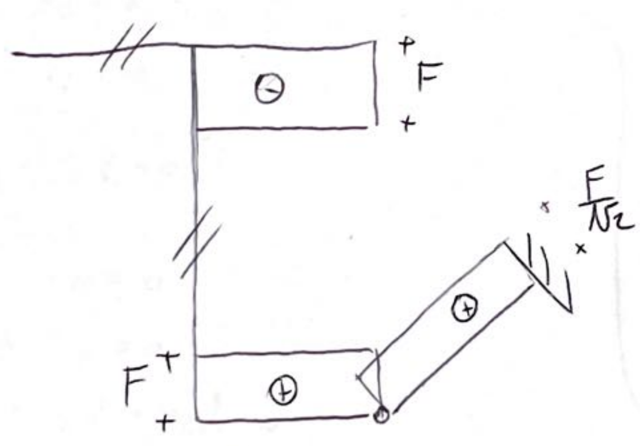
$$X_G = \frac{F}{\sqrt{2}} \; ; \; Y_G = \frac{F}{\sqrt{2}}$$

ANDANDO A DISEGNARE I DIAGRAMMI DELLE CdS:

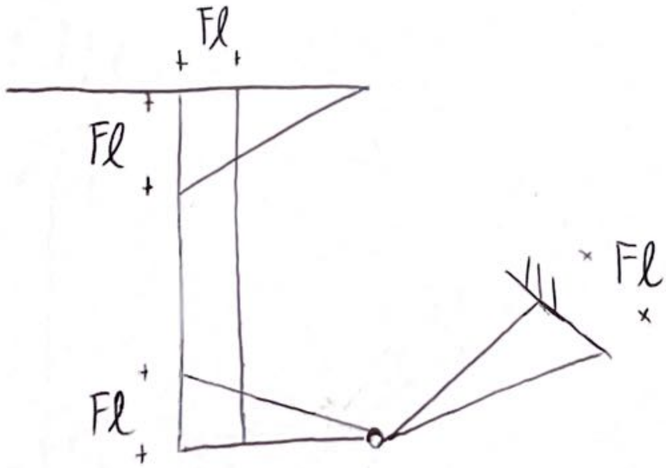
N.

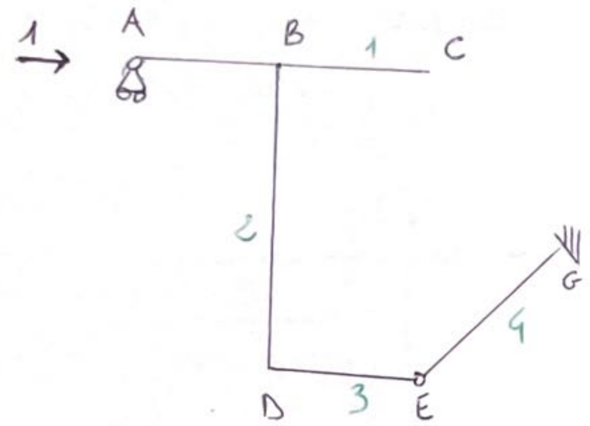


T.



M.





SISTEMA "1"

$$1\begin{cases}(\rightarrow) & 1 - X_B = 0 \\ (\uparrow) & Y_A - Y_B = 0 \\ (\overset{\curvearrowleft}{B}) & M_B - Y_A \ell = 0\end{cases}$$

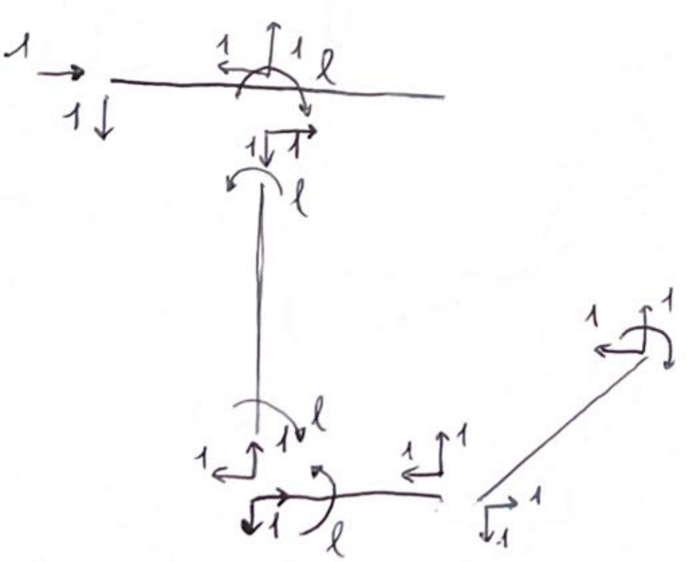
$$2\begin{cases}(\rightarrow) & X_B - X_D = 0 \\ (\uparrow) & Y_B - Y_D = 0 \\ (\overset{\curvearrowleft}{D}) & -M_B + M_D - 2\ell X_B = 0\end{cases}$$

$$3\begin{cases}(\rightarrow) & X_D - X_E = 0 \\ (\uparrow) & Y_D - Y_E = 0 \\ (\overset{\curvearrowleft}{D}) & -M_D - Y_E \ell = 0\end{cases}$$

$$4\begin{cases}(\rightarrow) & X_E + X_G = 0 \\ (\uparrow) & Y_E + Y_G = 0 \\ (\overset{\curvearrowleft}{G}) & M_G - Y_E \ell + X_E \ell = 0\end{cases}$$

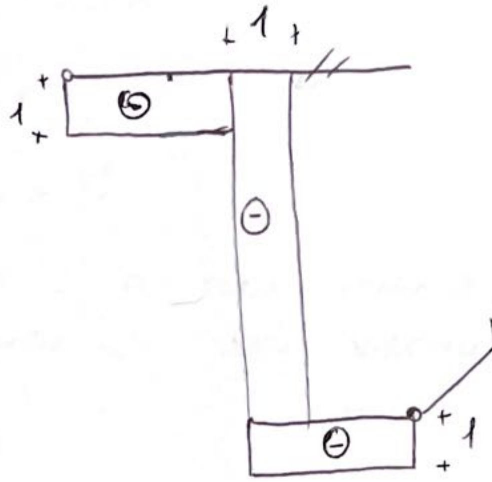
$$\begin{cases} Y_A = -1 \\ Y_B = -1 \\ Y_D = -1 \\ Y_E = -1 \\ Y_G = 1 \\ X_B = 1 \\ X_D = 1 \\ X_E = 1 \\ X_G = -1 \\ M_B = -\ell \\ M_D = \ell \\ M_G = -2\ell \end{cases}$$

IL DIAGRAMMA DI STRUTTURA LIBERA:

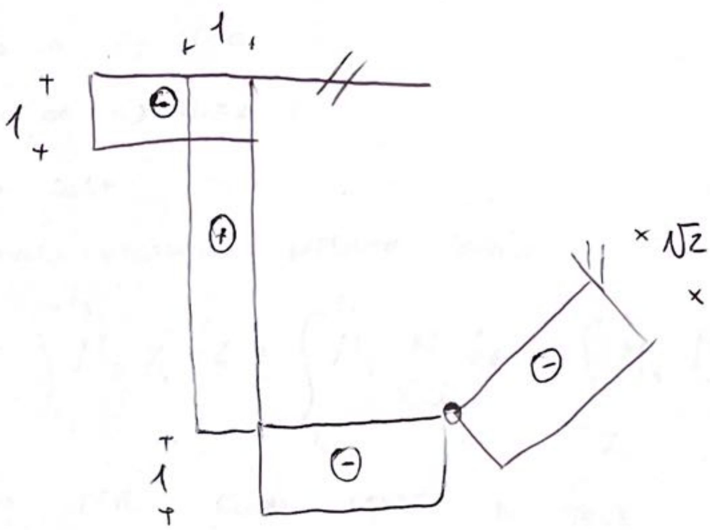


SUL VINCOLO IN G È POSSIBILE FARE LA SOMMA TRA X_G E Y_G E OTTENERE UNA REAZIONE $Y_G = \sqrt{2}$ DIRETTA PERPENDICOLARMENTE ALL'ASSE DELLA TRAVE.

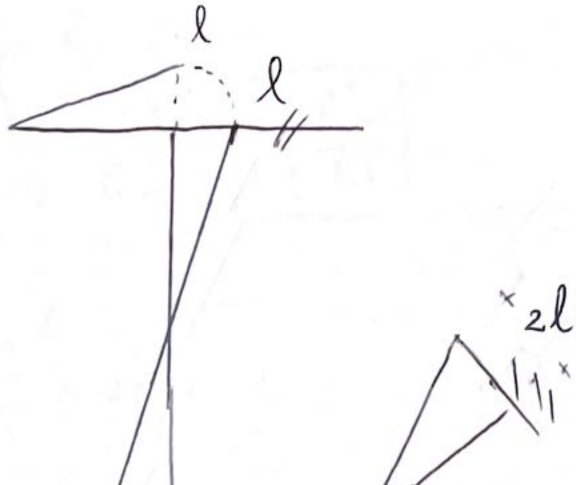
N_1



T_1



M_1



ANDANDO ORA A CONSIDERARE LE RELAZIONI CHE INTERCORRONO TRA IL SISTEMA EFFETTIVO, IL SISTEMA "0" E IL SISTEMA "1": (6)

$$M = M_0 + M_1 x$$
$$T = T_0 + T_1 x$$
$$N = N_0 + N_1 x$$

DOVE X E' LA NOSTRA INCOGNITA IPERSTATICA, PER TROVARLA SI USERA' IL PRINCIPIO DEI LAVORI VIRTUALI:

$$\mathcal{L}_v^E = 0$$

INTRODUCENDO LE IPOTESI:

$GA^* \to \infty \Rightarrow \gamma = 0$

$EA \to \infty \Rightarrow \varepsilon = 0$

$EI = cost$

IL LAVORO VIRTUALE INTERNO SARA':

$$\mathcal{L}_v^i = \int_{z_1}^{z_2} M_1 \chi \, dz = \int_{z_1}^{z_2} M_1 \frac{M}{EI} dz = \int_{z_1}^{z_2} M_1 \frac{M_0 + x M_1}{EI} dz$$

QUESTO PER OGNI TRATTO DI TRAVE:

AB: $$\int_0^{\ell} z \frac{zx}{EI} dz = \boxed{\frac{1}{3} \frac{\ell^3 x}{EI}}$$

BC: $$\int_0^{\ell} 0 \cdot dz = 0$$

BD: $$\int_0^{2\ell} (z-\ell) \frac{F\ell + xz - x\ell}{EI} dz = \frac{1}{EI} \int_0^{2\ell} F\ell z + x\ell z - xz^2 - F\ell^2 - x\ell^2 + xz\ell \, dz$$

BD:

$$\frac{1}{EI}\left(-\frac{8}{3}+2\right)x\ell^3 = \boxed{\frac{2}{3}\frac{x\ell^3}{EI}}$$

DE:

$$\int_0^\ell (\ell-z)\,\frac{F(z-\ell)+x(\ell-z)}{EI}\,dz = \frac{1}{EI}\left(\int_0^\ell F(\ell z-\ell^2-z^2+z\ell)+\right.$$

$$\left.+\,x(\ell^2+z^2-2\ell z)\,dz\right) = \frac{1}{EI}\left(\left(-F\ell^3\right)+\left(-\frac{F\ell^3}{3}\right)+F\ell^3+x\ell^3+\frac{x\ell^3}{3}-x\ell^3\right)=$$

$$= \boxed{\frac{1}{EI}\left(-\frac{F\ell^3}{3}+\frac{x\ell^3}{3}\right)}$$

EG:

$$\int_0^{\sqrt{2}\ell}\left(-\sqrt{2}z\right)\frac{\left(-\sqrt{2}z\,x+\frac{F}{\sqrt{2}}z\right)}{EI}\,dz = \frac{1}{EI}\int_0^{\sqrt{2}\ell} 2z^2x-Fz^2 = \boxed{\frac{\ell^3}{3EI}\left(4\sqrt{2}x-F2\sqrt{2}\right)}$$

PONENDO

$$\mathcal{L}_v^i = \mathcal{L}_v^E$$

$$\frac{1}{3}\ell^3\frac{x}{EI}+\frac{2}{3}\frac{x\ell^3}{EI}-\frac{F\ell^3}{3EI}+\frac{x\ell^3}{3EI}+\frac{\ell^3 4\sqrt{2}}{3EI}-\frac{2\sqrt{2}\ell^3}{3EI}F = 0$$

$$\frac{\ell^3}{EI}\left(\frac{4}{3}x+\frac{4\sqrt{2}}{3}x-\frac{1+2\sqrt{2}}{3}F\right)=0$$

$$(3+4\sqrt{2})x = (1+2\sqrt{2})F$$

$$\boxed{x = \frac{1+2\sqrt{2}}{4+4\sqrt{2}}F = \frac{3-\sqrt{2}}{4}F}$$

ANDANDO QUINDI A VALUTARE I DIAGRAMMI DELLE CdS: (8)

$N = N_0 + x N_1$

PER OGNI TRATTO:

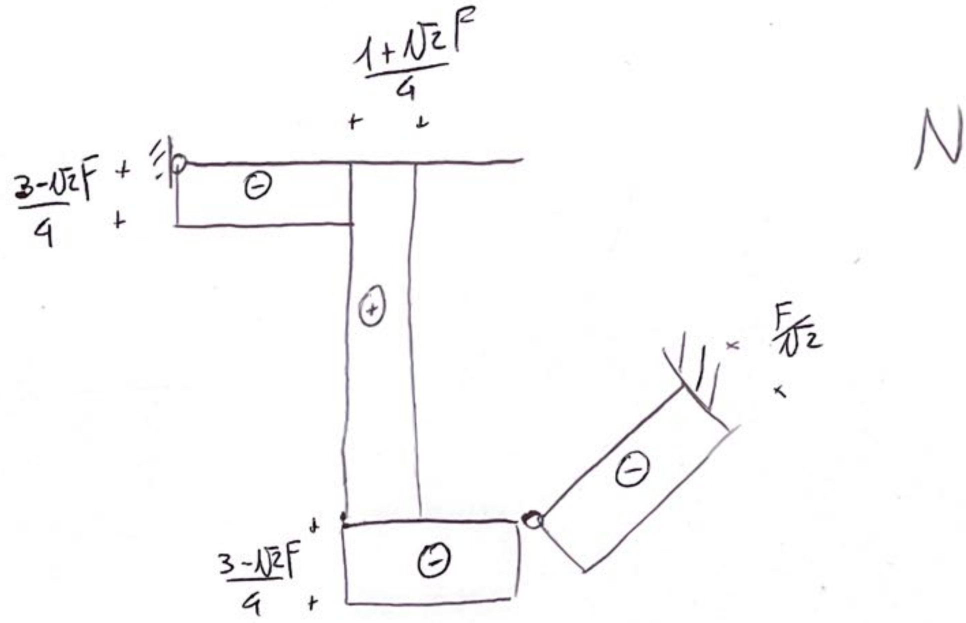
AB: $N = x$

BC: $N = 0$

BD: $N = F - x$

DE: $N = -x$

EG: $N = -\frac{F}{\sqrt{2}}$



$T = T_0 + x T_1$

PER OGNI TRATTO:

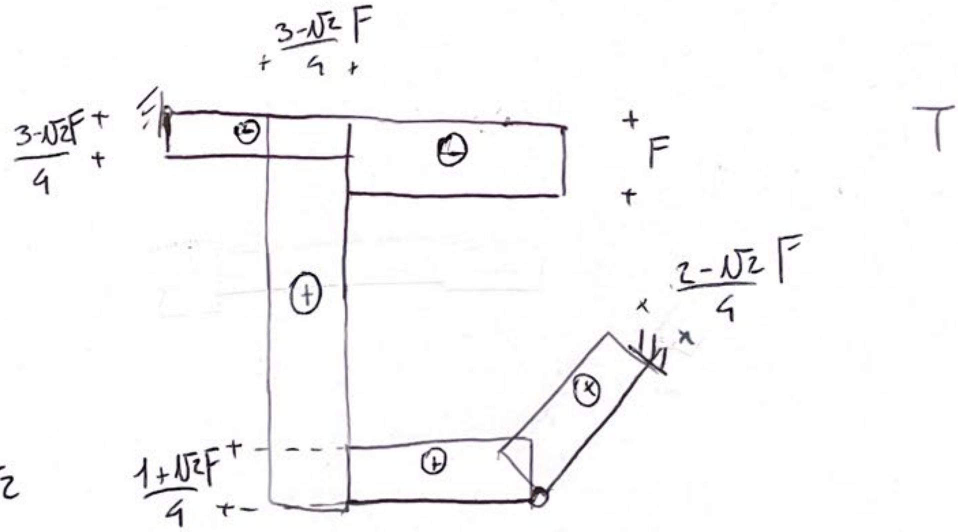
AB: $T = -x$

BC: $T = -F$

BD: $T = x$

DE: $T = F - x$

EG: $T = \frac{F}{\sqrt{2}} - x\sqrt{2}$



$M = M_0 + x M_1$

PER OGNI TRATTO

AB: $M = xz$

BC: $M = F(\ell - z)$

BD: $M = -F\ell + x(z-\ell)$

DE: $M = F(z-\ell) + x(\ell - z)$

EG: $M = \frac{Fz}{\sqrt{2}} - xz\sqrt{2}$