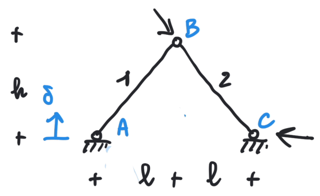
ESERCIZI SVOLTI (METODO ANALITICO)

ES 1 2 corpi rigidi



ARCO A TRE CERNIERE

Corpo 1
$B \equiv (l, h)$

$\rightarrow u_{B_1} = u_A - \vartheta_1 h$
$v_{B_1} = v_A + \vartheta_1 l$

Corpo 2
$B \equiv (-l, h)$
$u_{B_2} = u_C - \vartheta_2 h$
$v_{B_2} = v_C - \vartheta_2 l$

$u = u_A - \vartheta_1 y_1$ $\quad$ $u = u_C - \vartheta_2 y_2$
$v = v_A + \vartheta_1 x_1$ $\quad$ $v = v_C + \vartheta_2 x_2$

$\underline{q} = [u_A \ v_A \ \vartheta_1 \ u_C \ v_C \ \vartheta_2]^T$ 6 incognite

$u_A = 0$ $\quad$ $v_A = \delta$ $\quad$ $u_A - \vartheta_1 h - u_C + \vartheta_2 h = 0$ $\quad$ $u_C = 0$ $\quad$ $v_C = 0$

6 eq.ni $\quad$ $v_A + \vartheta_1 l - v_C + \vartheta_2 l = 0$

$\underline{A}\underline{q} = \underline{s}$ $\quad$ A $m = 6$ righe $n = 6$ colonne
quadrata

$\underline{q}$ $\underline{s}$ 6 componenti
vettori colonna

$$\underline{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -h & -1 & 0 & h \\ 0 & 1 & l & 0 & -1 & l \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \underline{s} = \begin{bmatrix} 0 \\ \delta \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$=0$
$=0$

Per classificare la strutt.
è suff. conoscere $\underline{A}$.

$\rho = \text{rango } \underline{A} \stackrel{?}{=} m = n = 6$

$\det \underline{A} = 1 \cdot (-1)^{1+1} \det \underline{M}_{11}(\underline{A})$ regola di Laplace

$\det \underline{M}_{11}(\underline{A}) = 1 \cdot (-1)^{5+4} \det \underline{M}_{54}(\underline{M}_{11}(\underline{A}))$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -h & 1 & h \\ 1 & l & 0 & l \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= 1 \cdot (-1)^{1+1} \det \begin{bmatrix} -h & 1 & h \\ l & 0 & l \\ 0 & 1 & 0 \end{bmatrix} \begin{matrix} -h & 1 \\ l & 0 \\ 0 & 1 \end{matrix}$$

(SARRUS)

$= hl - (-hl)$
$= 2hl \neq 0$ (*)
se $h \neq 0$ e $l \neq 0$

$\det A =$ prodotto tra fattori $\neq 0$

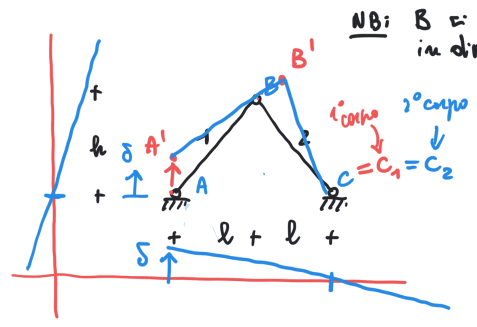
$\Rightarrow \det A \neq 0$!

$\Rightarrow$ Sistema **ISOCINEMATICO**

Risolviamo il sistema $\underline{A}\underline{q} = \underline{s}$
risolvere per sostituz.

Svolgendo i calcoli si trova

$u_A = 0$
$v_A = \delta$
$\vartheta_1 = -\delta/2l$
$\vartheta_2 = -\delta/2l$
$u_C = 0$
$v_C = 0$



NB: B si sposta in direz. perp. a $\underline{BC}$

$\vartheta_1 = \vartheta_2$
$C_1 = C_2$
$\Downarrow$
i due corpi si spostano in modo SOLIDALE

Costruzione grafica del campo di spostamento

rif. corpo 1.

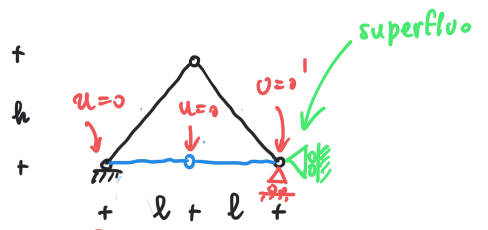
Centro C_1 $\quad$ $x_{C_1} = -\frac{v_A}{\vartheta_1} = -\frac{\delta}{-\delta/2l} = 2l \Rightarrow C_1 \equiv (2l, 0)$

$y_{C_1} = \frac{u_A}{\vartheta_1} = 0$

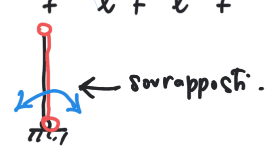
$\underline{\text{OSS}}$ importante.

$\det A = 0$ se $h = 0$ oppure $l = 0$ (vedi (*))

$h = 0$

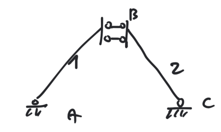


$l = 0$



TRE CERNIERE ALLINEATE $\rightarrow$ ARCO DEGENERE

Variante dell'esercizio



Vogliamo solamente classificare la struttura
$\rightarrow \delta = 0$

cerniera

$u_A = 0$ $\quad$ $v_A = \delta$ $\quad$ $u_A - \vartheta_1 h - u_C + \vartheta_2 h = 0$ $\quad$ $u_C = 0$ $\quad$ $v_C = 0$

$\vartheta_2 - \vartheta_1 = 0$

$\underline{\text{Vincolo in B}}$ $\begin{cases} \vartheta_2 = \vartheta_1 \\ u_{B2} = u_{B1} \end{cases}$ $\leftarrow$ direttamente in termini delle componenti di $\underline{q}$.

1ª eq.ne cerniera! invariata

$$\underline{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -h & -1 & 0 & h \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad q = \begin{bmatrix} \cdot \\ \cdot \\ \vartheta_1 \\ \cdot \\ \cdot \\ \vartheta_2 \end{bmatrix}$$

Si trova: $\det \underline{A} = 0 \Rightarrow$ struttura DEGENERE