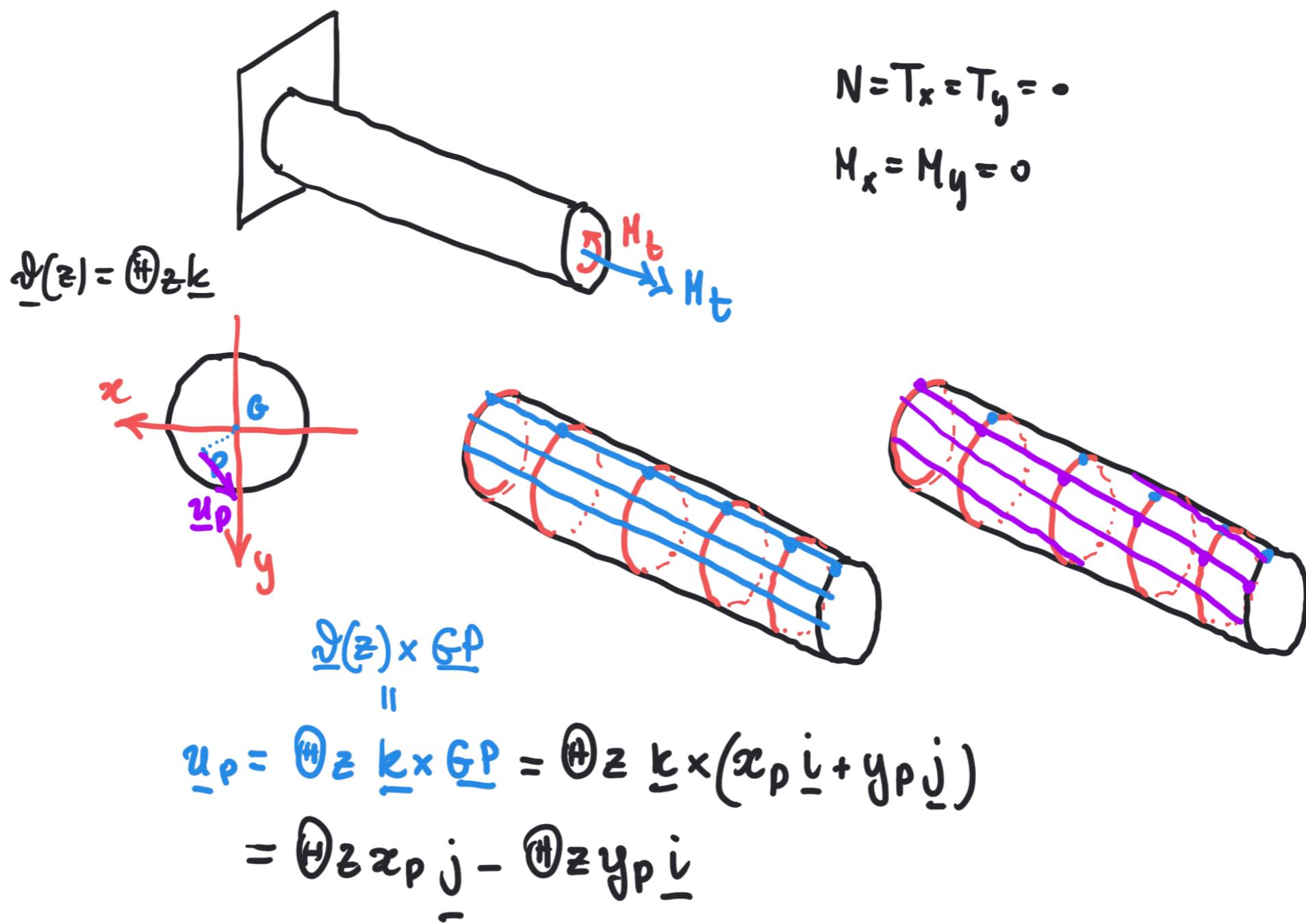
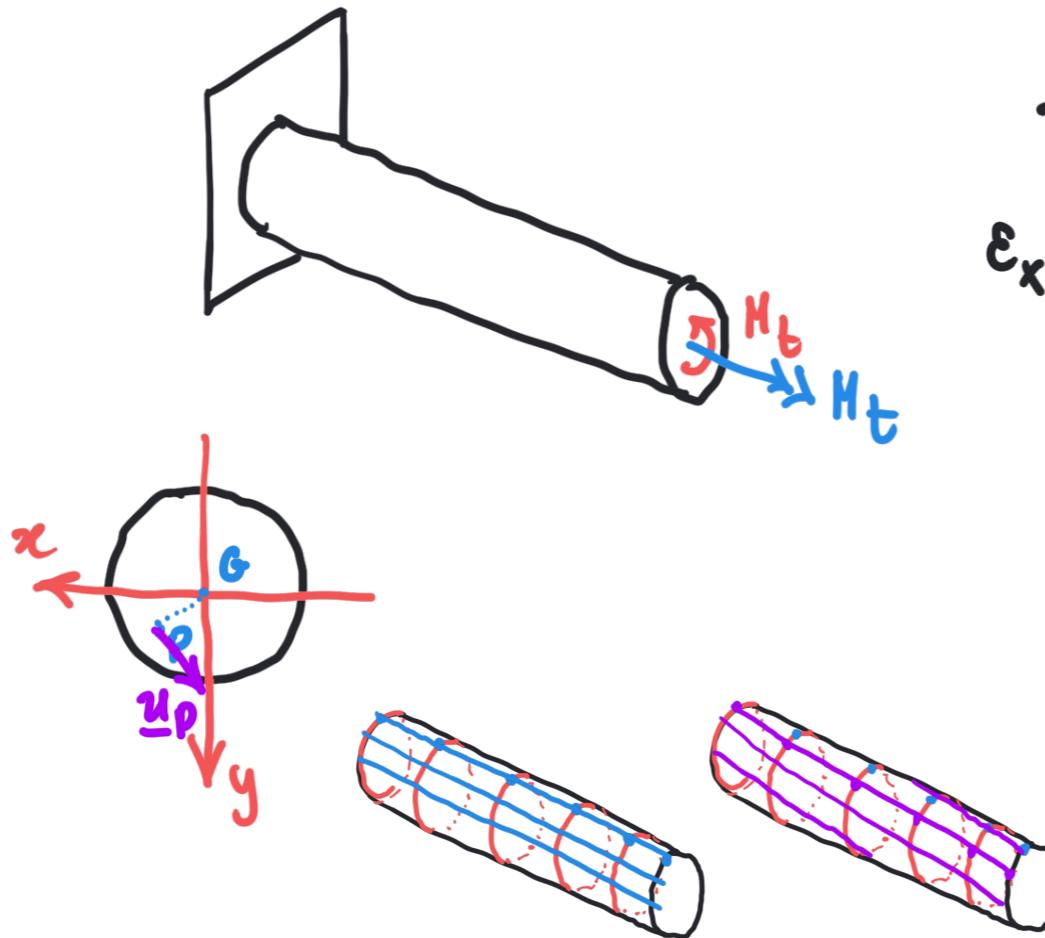


Ip. circa: ogni sezione ruota attorno all'asse z.
in misura proporzionale alle coordinate z.

Torsione uniforme



Torsione uniforme



$$u(x, y, z) = -\Theta z y$$

$$v(x, y, z) = \Theta z x$$

$$w(x, y, z) = 0$$

$$\epsilon_x = \frac{\partial u}{\partial x} = 0 \quad \epsilon_y = 0 \quad \epsilon_z = 0$$

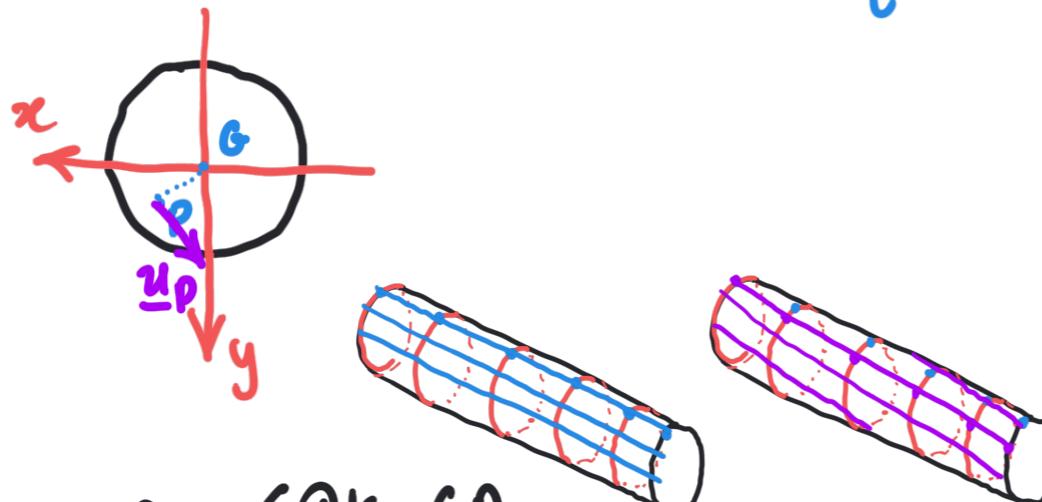
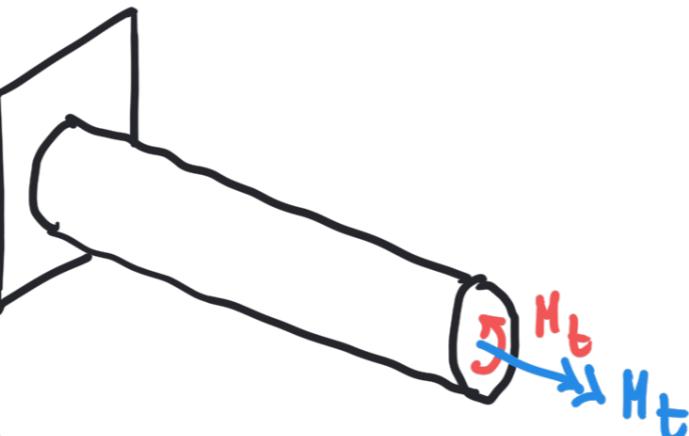
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$$

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = -\Theta y$$

$$\gamma_{zy} = \Theta x$$

$$\begin{aligned} \underline{u}_p &= \Theta z \underline{k} \times \underline{GP} = \Theta z \underline{k} \times (x_p \dot{\underline{i}} + y_p \dot{\underline{j}}) \\ &= \Theta z x_p \dot{\underline{j}} - \Theta z y_p \dot{\underline{i}} \end{aligned}$$

Torsione uniforme



$$\underline{\tau}_P = G \Theta \underline{k} \times \underline{G P}$$

$$\underline{u}_P = \Theta z \underline{k} \times \underline{G P} \quad , \quad \underline{G P}$$

$$\underline{\tau}_P = G \Theta \underline{k} \times (\underline{x_P i} + \underline{y_P j})$$

$$u(x, y, z) = -\Theta z y$$

$$v(x, y, z) = \Theta z x$$

$$w(x, y, z) = 0$$

$$\epsilon_x = \frac{\partial u}{\partial x} = 0 \quad \epsilon_y = 0 \quad \epsilon_z = 0$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$$

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = -\Theta y$$

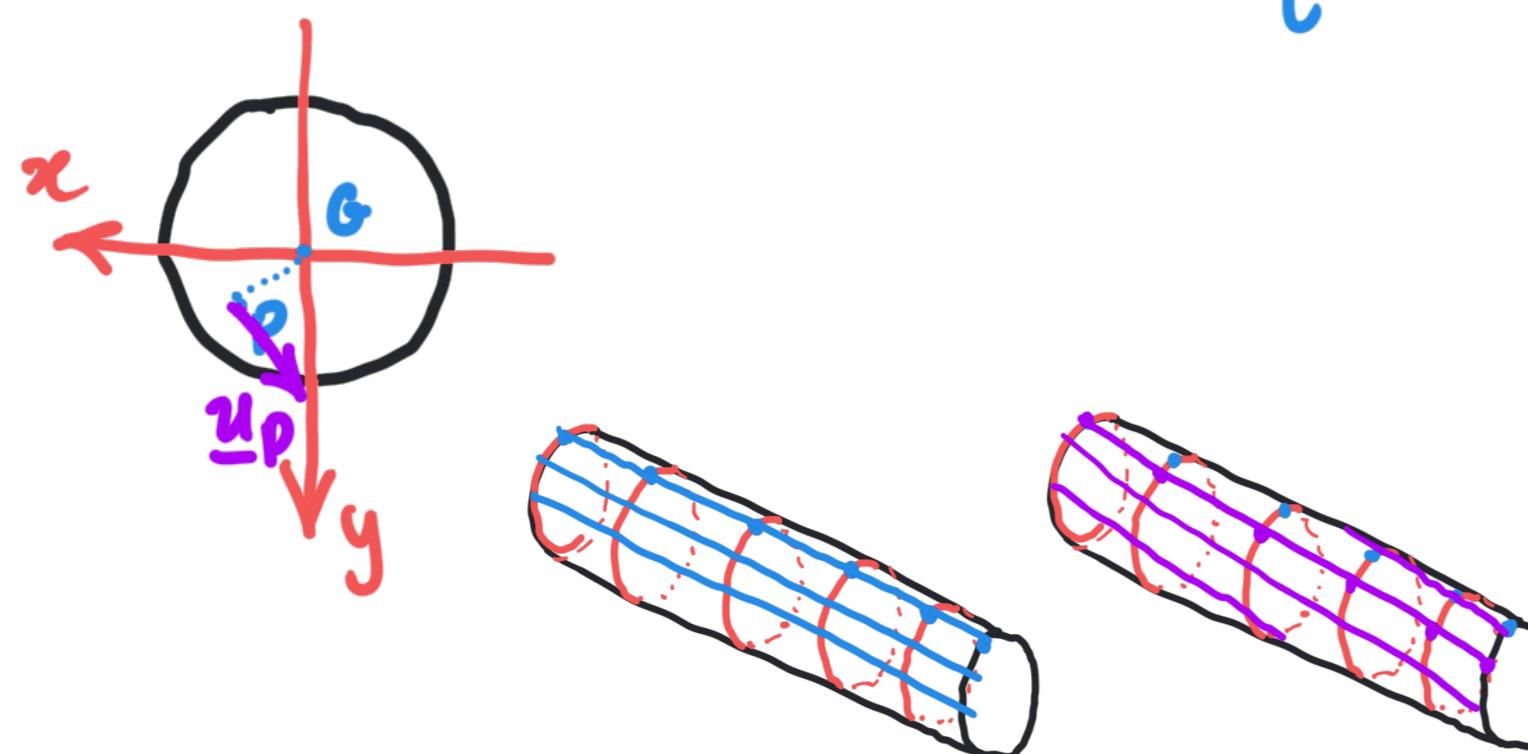
$$\gamma_{zy} = \Theta x$$

$$\sigma_x = \tau_{yz} = \tau_{zx} = 0 \quad \tau_{xy} = 0$$

$$\tau_{zx} = -G \Theta y \quad \tau_{zy} = G \Theta x$$

$$\begin{aligned} \underline{\tau} &= \tau_{zx} \underline{i} + \tau_{zy} \underline{j} = -G \Theta y \underline{i} + G \Theta x \underline{j} \\ &= G \Theta (\underline{x j} - \underline{y i}) = G \Theta \underline{k} \times (\underline{x i} + \underline{y j}) \end{aligned}$$

Torsione uniforme



$$\underline{u}_P = \Theta z \underline{k} \times \underline{GP}$$

$$\underline{\tau}_P = G\Theta \underline{k} \times \underline{GP}$$

$$N = 0 \quad M_x = M_y = 0$$

$$T_x = \int c_{zx} dA = -G\Theta S_x = 0$$

$$T_y = 0$$

$$M_t = \int (c_{zy} x - c_{zx} y) dA$$

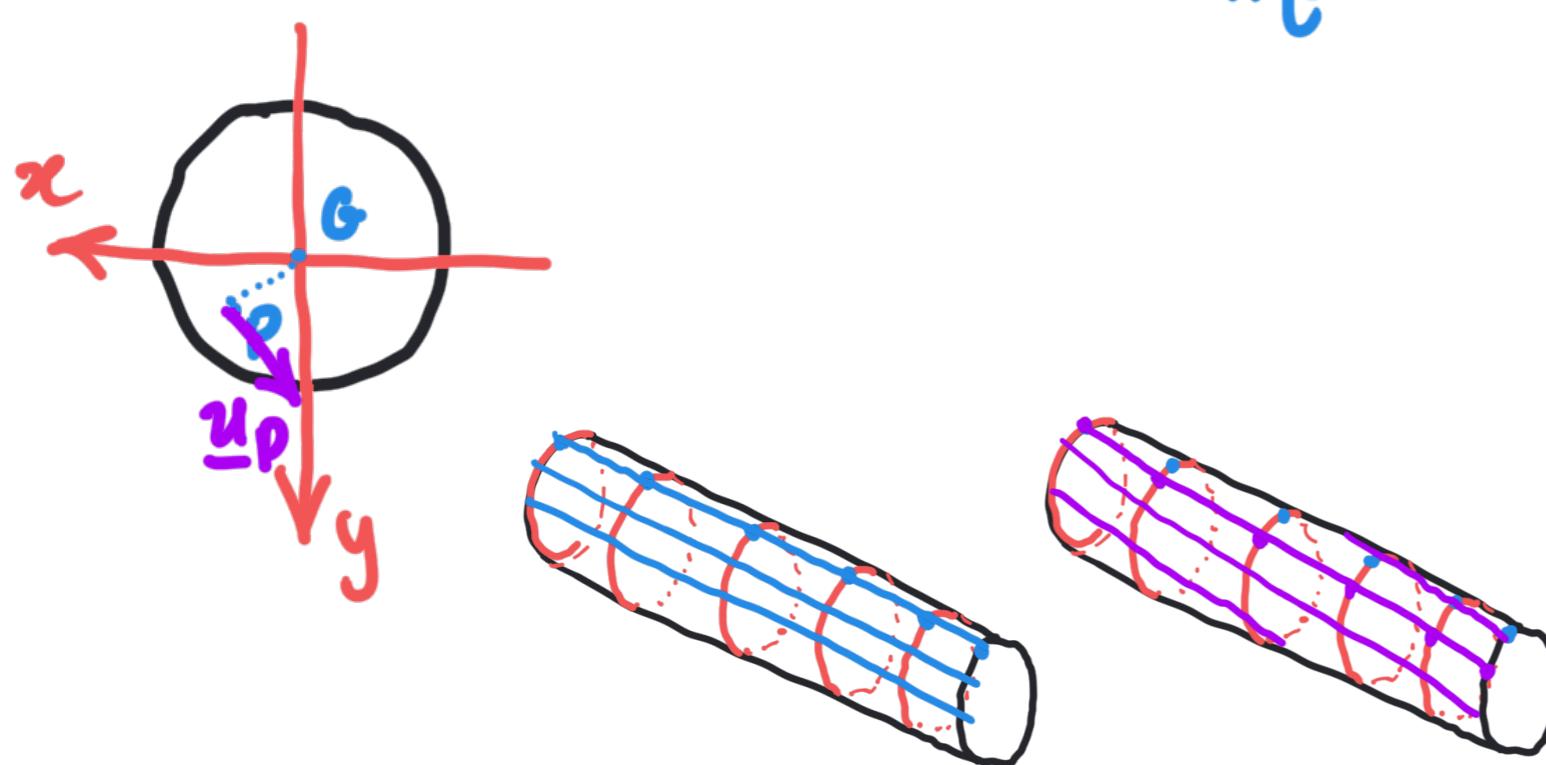
$$= G\Theta \int (a^2 + y^2) dA = G\Theta I_p$$

$$\sigma_x = \sigma_y = \sigma_z = 0 \quad \tau_{xy} = 0$$

$$\tau_{zx} = -G\Theta y \quad \tau_{zy} = G\Theta x$$

$$\begin{aligned} \underline{\tau} &= \tau_{zx} \underline{i} + \tau_{zy} \underline{j} = -G\Theta y \underline{i} + G\Theta x \underline{j} \\ &= G\Theta (x \underline{j} - y \underline{i}) = G\Theta \underline{k} \times (x \underline{i} + y \underline{j}) \end{aligned}$$

Torsione uniforme



$$\underline{u}_p = \Theta z \underline{k} \times \underline{GP}$$

$$\underline{\epsilon}_p = G \Theta \underline{k} \times \underline{GP}$$

$$\Theta = \frac{M_t}{G I_p}$$

$$N = 0 \quad M_x = M_y = 0$$

$$T_x = \int_A c_{zx} dA = -G\Theta S_y = 0$$

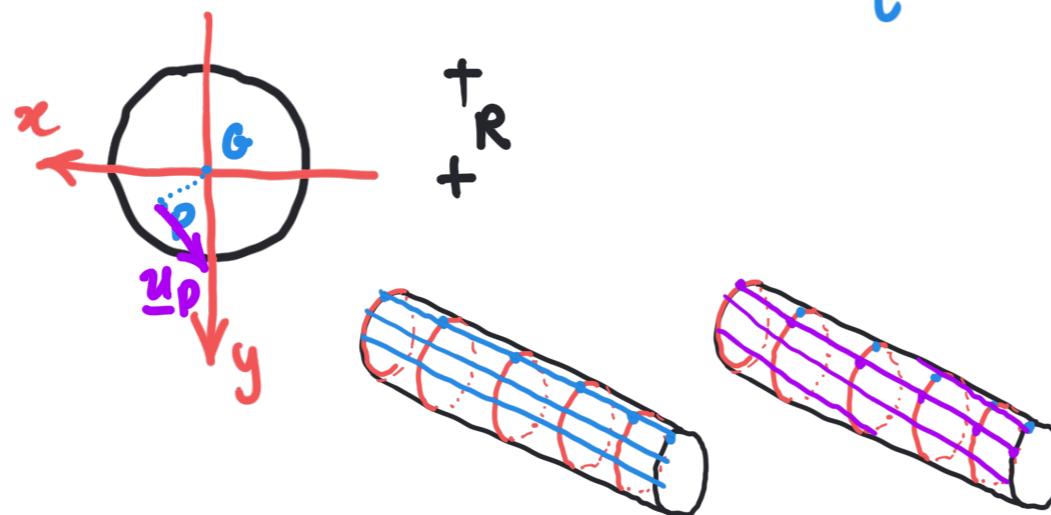
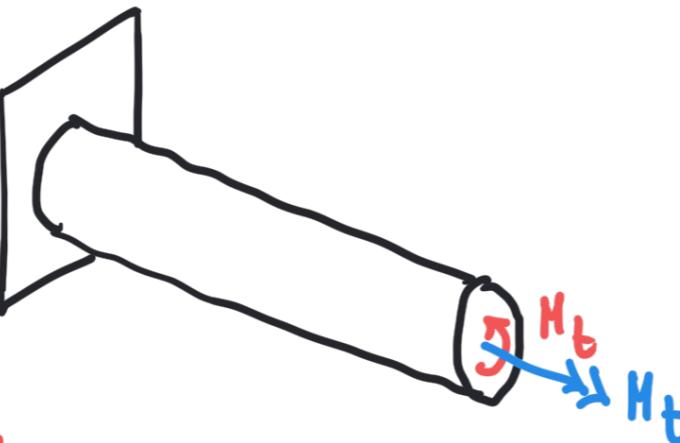
$$T_y = 0$$

$$M_t = \int_A (c_{zy} x - c_{zx} y) dA$$

$$= G\Theta \int_A (x^2 + y^2) dA = G\Theta I_p$$

Sea. circ. $I_p = \frac{\pi R^4}{2}$

TORSIONE UNIFORME



disegnando originale

$$\tau(r) = G \theta r$$

$$= \frac{M_t}{I_p} r$$

$$= \frac{2 M_t}{\pi R^4} r$$

$$M_t = \int_A (\tau_{zy} x - \tau_{xz} y) dA$$

$$= G \theta \int_A (x^2 + y^2) dA = G \theta I_p$$

Sez. cir. $I_p = \frac{\pi R^4}{2}$

$$u_p = \underline{\theta} \underline{z} \underline{k} \times \underline{G} \underline{P}$$

$$\underline{\tau}_p = G \underline{\theta} \underline{k} \times \underline{G} \underline{P}$$

$$\underline{\theta} = \frac{M_t}{G I_p}$$

$$\tau_{max} = \frac{2 M_t}{\pi R^3}$$



$$R = \frac{R_e + R_i}{2}$$

$$S = R_e - R_i$$

$$S \ll R$$

$$I_p = \pi \frac{R_e^4 - R_i^4}{2} \approx 2\pi R^3 S$$

$$R_e^4 - R_i^4 = \underbrace{(R_e^2 + R_i^2)}_{(R_e + R_i)(R_e - R_i)} \underbrace{(R_e^2 - R_i^2)}_{\approx 4R^3 S}$$

$$\underline{u}_p = \Theta \underline{z} \times \underline{k} \times \underline{G} \underline{P}$$

$$\underline{\tau}_p = G \Theta \underline{k} \times \underline{G} \underline{P}$$

$$\Theta = \frac{M_t}{G I_p}$$

$$\begin{aligned} \underline{\tau}(r) &= G \Theta \underline{r} \\ &= \frac{M_t}{I_p} \underline{r} \\ &= \frac{2M_t}{\pi R^4} \underline{r} \end{aligned}$$

$$\begin{aligned} M_t &= \int (\tau_{zy} x - \tau_{zx} y) dA \\ &= G \Theta \int (x^2 + y^2) dA = G \Theta I_p \end{aligned}$$

$$\text{Sol. curr. } I_p = \frac{\pi R^4}{2}$$

$$\tau_{zx} = -G \Theta y \quad \tau_{zy} = G \Theta x$$

$$\tau_{max} = \frac{2M_t}{\pi R^3}$$