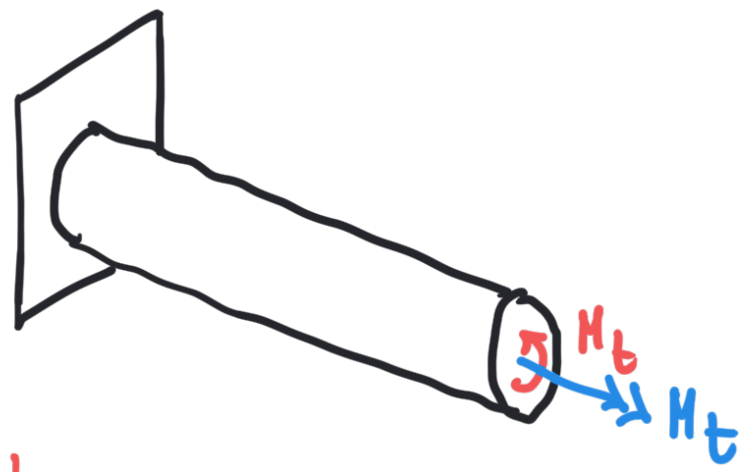
Ip. cinem.: ogni sezione ruota attorno all'asse z
in misura proporzionale alla coordinata z.

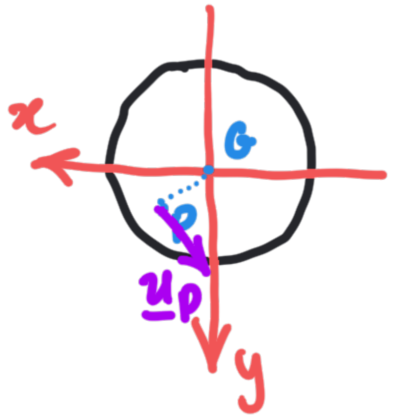
Torsione uniforme

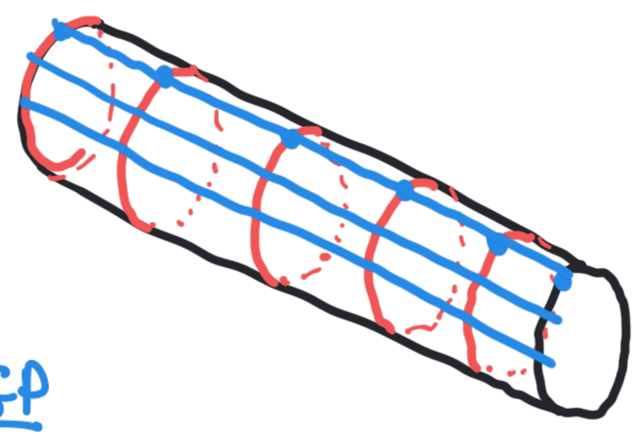


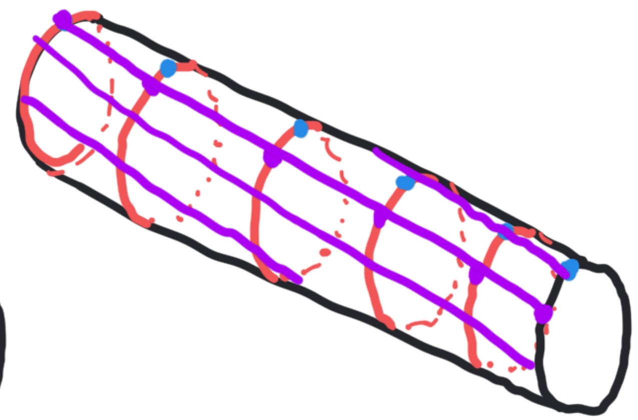
$N = T_x = T_y = 0$

$M_x = M_y = 0$

$\underline{\vartheta}(z) = \Theta z \underline{k}$







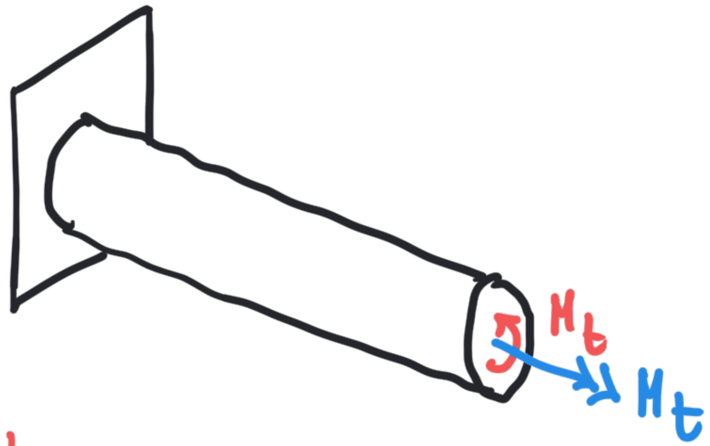
$$\underline{\vartheta}(z) \times \underline{GP}$$

$$\parallel$$

$$\underline{u}_P = \Theta z\, \underline{k} \times \underline{GP} = \Theta z\, \underline{k} \times (x_P \underline{i} + y_P \underline{j})$$

$$= \Theta z\, x_P \underline{j} - \Theta z\, y_P \underline{i}$$

Torsione uniforme



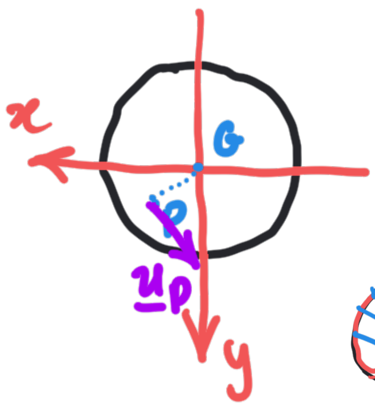
$$u(x,y,z) = -\Theta z y$$
$$v(x,y,z) = \Theta z x$$
$$w(x,y,z) = 0$$

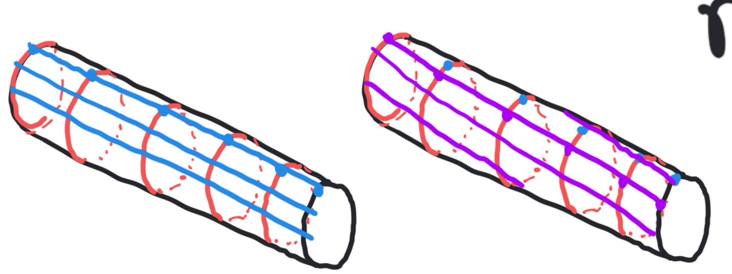
$$\varepsilon_x = \frac{\partial u}{\partial x} = 0 \qquad \varepsilon_y = 0 \qquad \varepsilon_z = 0$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$$

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = -\Theta y$$

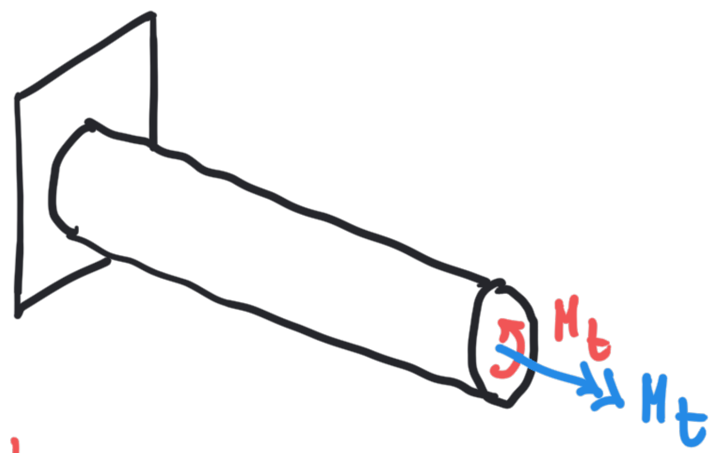
$$\gamma_{zy} = \Theta x$$

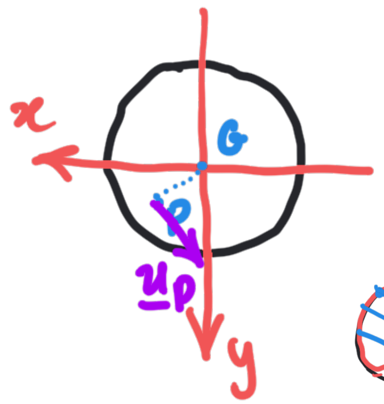


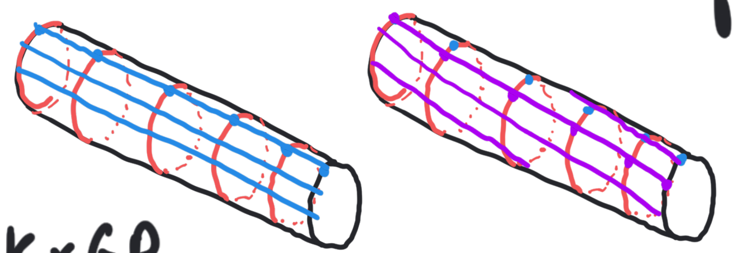


$$\underline{u}_P = \Theta z\, \underline{k} \times \underline{GP} = \Theta z\, \underline{k} \times (x_P\, \underline{i} + y_P\, \underline{j})$$
$$= \Theta z x_P\, \underline{j} - \Theta z y_P\, \underline{i}$$

Torsione uniforme







$$u(x,y,z) = -\Theta z y$$

$$v(x,y,z) = \Theta z x$$

$$w(x,y,z) = 0$$

$$\varepsilon_x = \frac{\partial u}{\partial x} = 0 \qquad \varepsilon_y = 0 \qquad \varepsilon_z = 0$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$$

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = -\Theta y$$

$$\gamma_{zy} = \Theta x$$

$$\sigma_x = \sigma_y = \sigma_z = 0 \quad \tau_{xy} = 0$$

$$\tau_{zx} = -G\Theta y \quad \tau_{zy} = G\Theta x$$

$$\underline{\tau} = \tau_{zx}\underline{i} + \tau_{zy}\underline{j} = -G\Theta y\,\underline{i} + G\Theta x\,\underline{j}$$

$$= G\Theta(x\underline{j} - y\underline{i}) = G\Theta\underline{k}\times(x\underline{i} + y\underline{j})$$

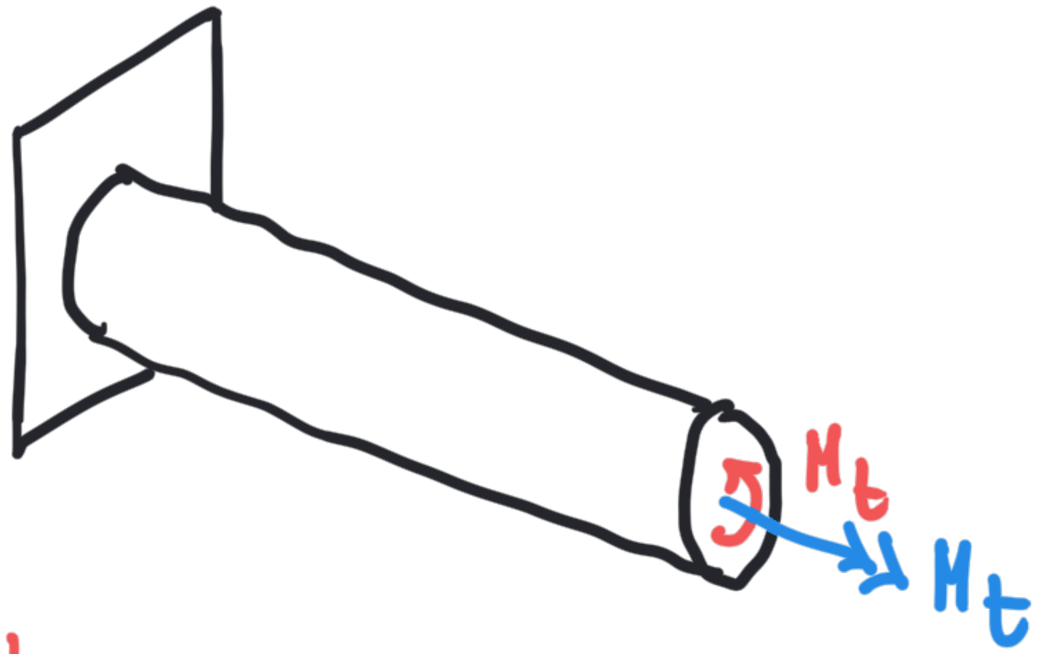
$$\underline{\tau}_P = G\Theta\underline{k}\times\underline{GP}$$

$$\underline{u}_P = \Theta z\,\underline{k}\times\underline{GP}$$

$$= \underline{GP}$$

$$\underline{\tau}_P = G\Theta\underline{k}\times(x_P\underline{i} + y_P\underline{j}$$

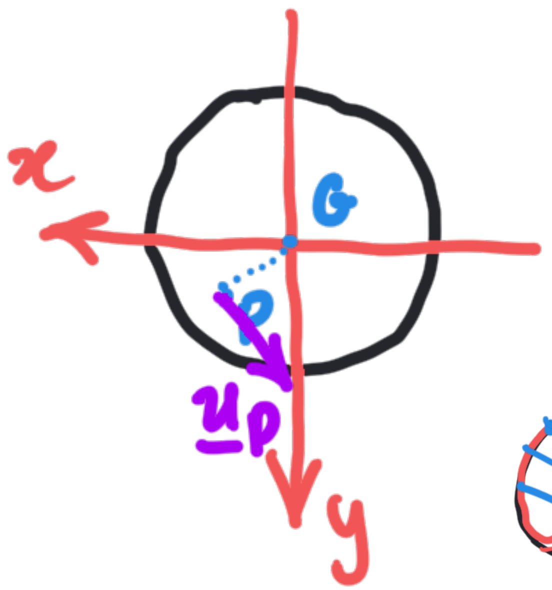
Torsione uniforme



$$N = 0 \qquad M_x = M_y = 0$$

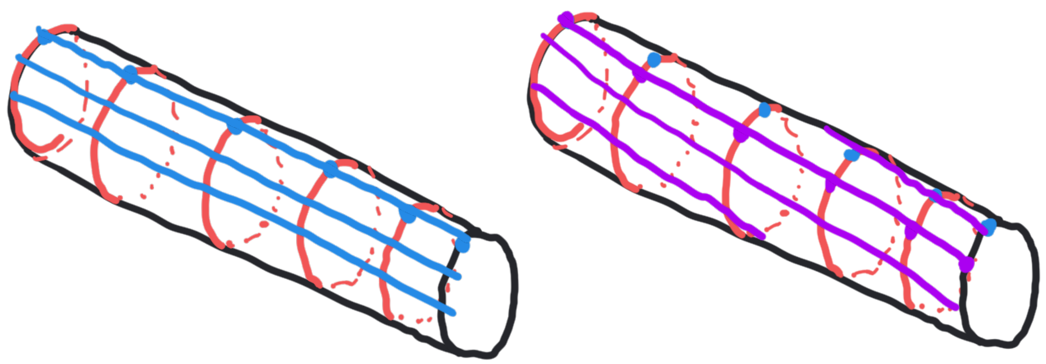
$$T_x = \int_A \tau_{zx}\, dA = -G\Theta S_x = 0$$

$$T_y = 0$$



$$M_t = \int_A (\tau_{zy}\, x - \tau_{zx}\, y)\, dA$$

$$= G\Theta \int_A (x^2 + y^2)\, dA = G\Theta I_p$$



$$\sigma_x = \sigma_y = \sigma_z = 0 \qquad \tau_{xy} = 0$$

$$\tau_{zx} = -G\Theta y \qquad \tau_{zy} = G\Theta x$$

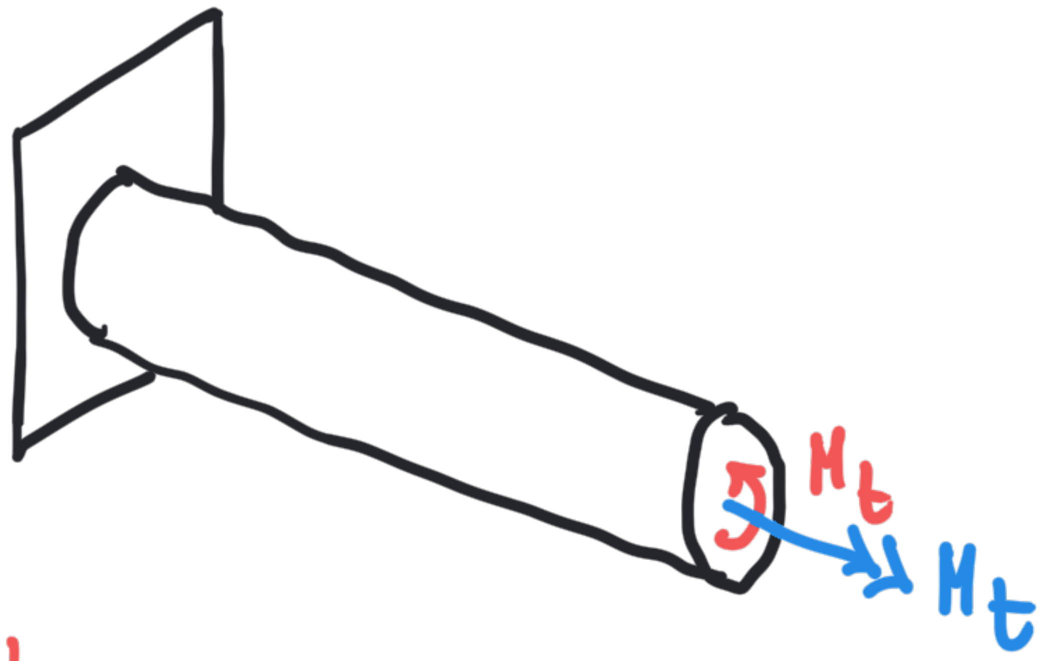
$$\underline{\tau} = \tau_{zx}\, \underline{i} + \tau_{zy}\, \underline{j} = -G\Theta y\, \underline{i} + G\Theta x\, \underline{j}$$

$$= G\Theta (x\, \underline{j} - y\, \underline{i}) = G\Theta\, \underline{k} \times (x\, \underline{i} + y\, \underline{j})$$

$$\underline{u}_P = \Theta z\, \underline{k} \times \underline{GP}$$

$$\underline{\tau}_P = G\Theta\, \underline{k} \times \underline{GP}$$

Torsione uniforme



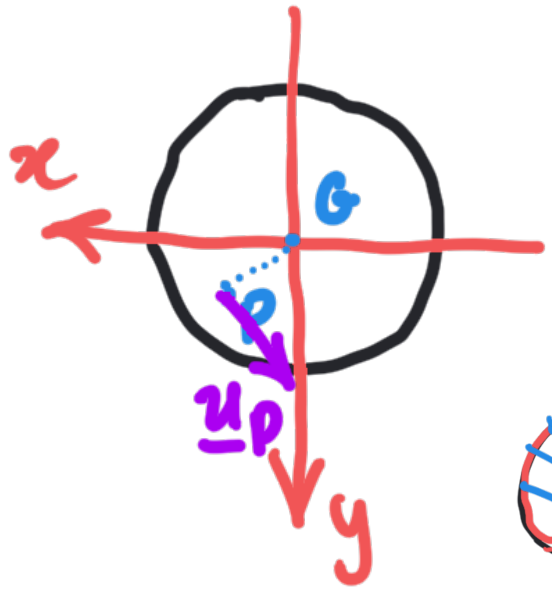
$N = 0 \qquad M_x = M_y = 0$

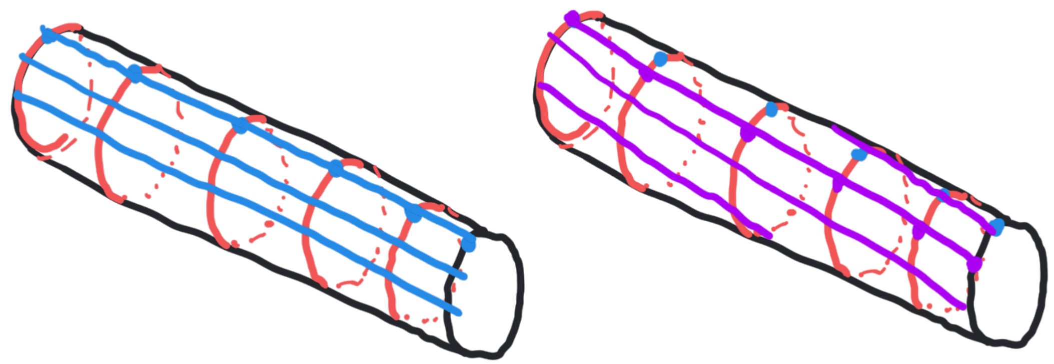
$$T_x = \int_A \tau_{zx}\, dA = -G\Theta S_y = 0$$

$$T_y = 0$$

$$M_t = \int_A (\tau_{zy}\, x - \tau_{zx}\, y)\, dA$$

$$= G\Theta \int_A (x^2 + y^2)\, dA = G\Theta I_p$$



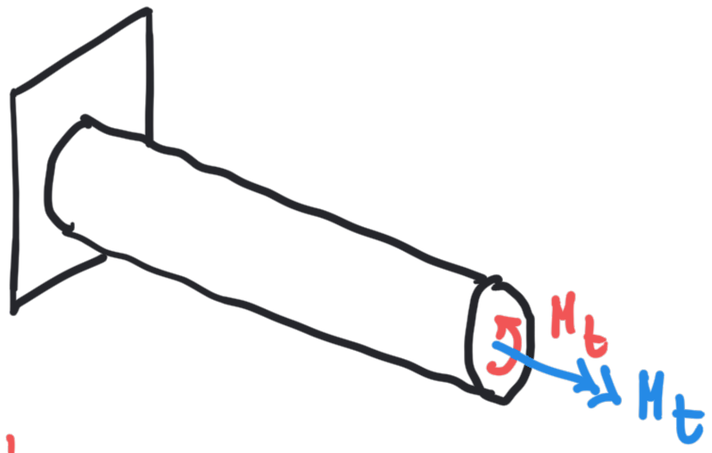


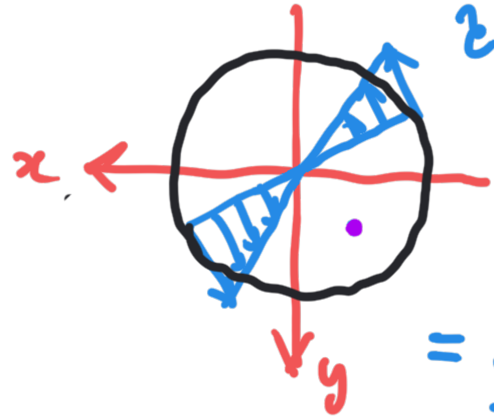
sez. circ. $\quad I_p = \dfrac{\pi R^4}{2}$

$$\underline{u}_P = \Theta z\, \underline{k} \times \underline{GP}$$

$$\underline{\tau}_P = G\Theta\, \underline{k} \times \underline{GP} \qquad \Theta = \frac{M_t}{G I_p}$$

Torsione uniforme



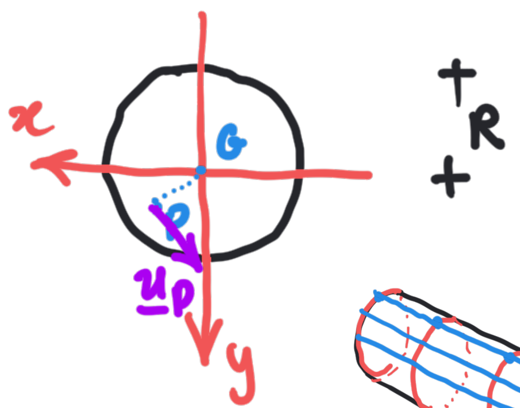


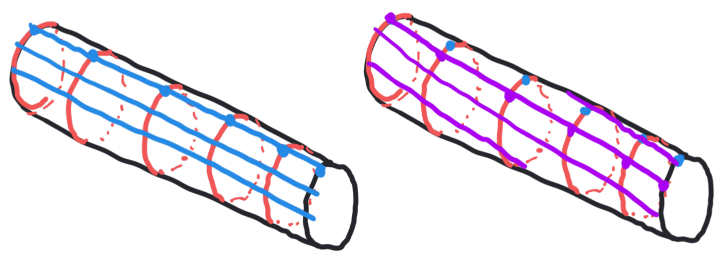
distanza da origine

$$\tau(r) = G\Theta\, r = \frac{M_t}{I_p}\, r = \frac{2M_t}{\pi R^4}\, r$$

$$M_t = \int_A (\tau_{zy}\, x - \tau_{zx}\, y)\, dA = G\Theta \int_A (x^2+y^2)\, dA = G\Theta I_p$$

sez. circ. $I_p = \dfrac{\pi R^4}{2}$





$$\underline{u}_P = \Theta z\, \underline{k} \times \underline{GP}$$

$$\underline{\tau}_P = G\Theta\, \underline{k} \times \underline{GP} \qquad \Theta = \frac{M_t}{G I_p} \qquad \tau_{max} = \frac{2M_t}{\pi R^3}$$

$$R = \frac{R_e + R_i}{2}$$

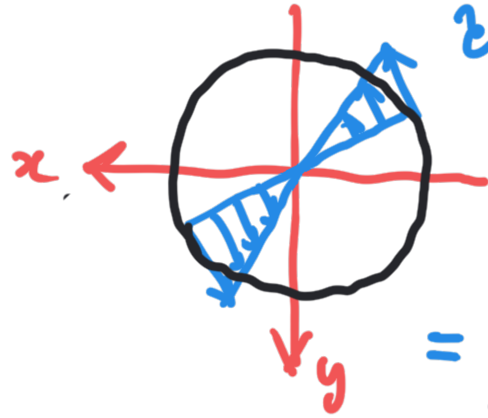
$$s = R_e - R_i$$

$s \ll R$

$$I_p = \pi \frac{R_e^4 - R_i^4}{2} \simeq 2\pi R^3 s$$

$$R_e^4 - R_i^4 = (R_e^2 + R_i^2)\underbrace{(R_e^2 - R_i^2)}_{(R_e + R_i)(R_e - R_i)}$$

$$\simeq 4R^3 s$$



$$\tau(r) = G\Theta r = \frac{M_t}{I_p} r = \frac{2M_t}{\pi R^4} r$$

$$M_t = \int_A (\tau_{zy} x - \tau_{zx} y)\, dA = G\Theta \int_A (x^2 + y^2)\, dA = G\Theta I_p$$

sez. circ. $I_p = \frac{\pi R^4}{2}$

$\tau_{zx} = -G\Theta y \quad \tau_{zy} = G\Theta x$

$$\underline{u}_P = \Theta z \, \underline{k} \times \underline{GP}$$

$$\underline{\tau}_P = G\Theta \, \underline{k} \times \underline{GP} \qquad \Theta = \frac{M_t}{G I_p}$$

$$\tau_{max} = \frac{2M_t}{\pi R^3}$$