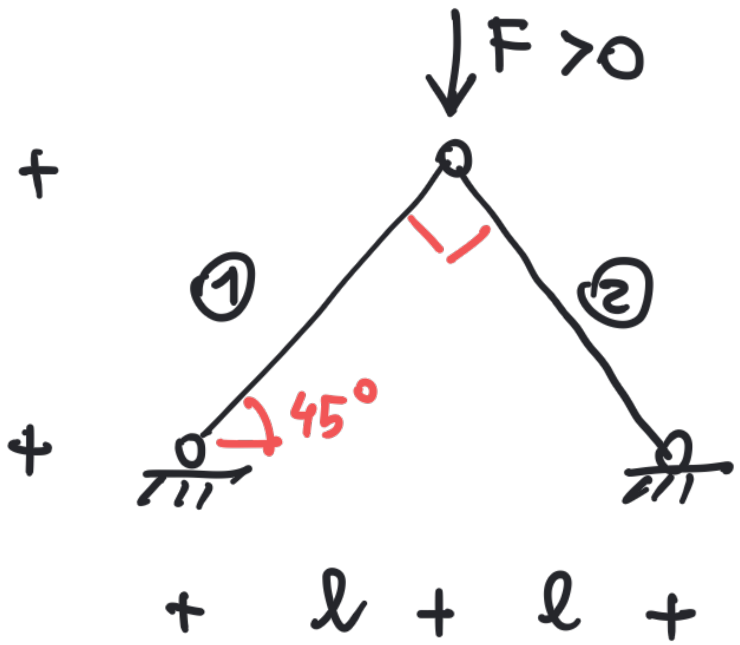
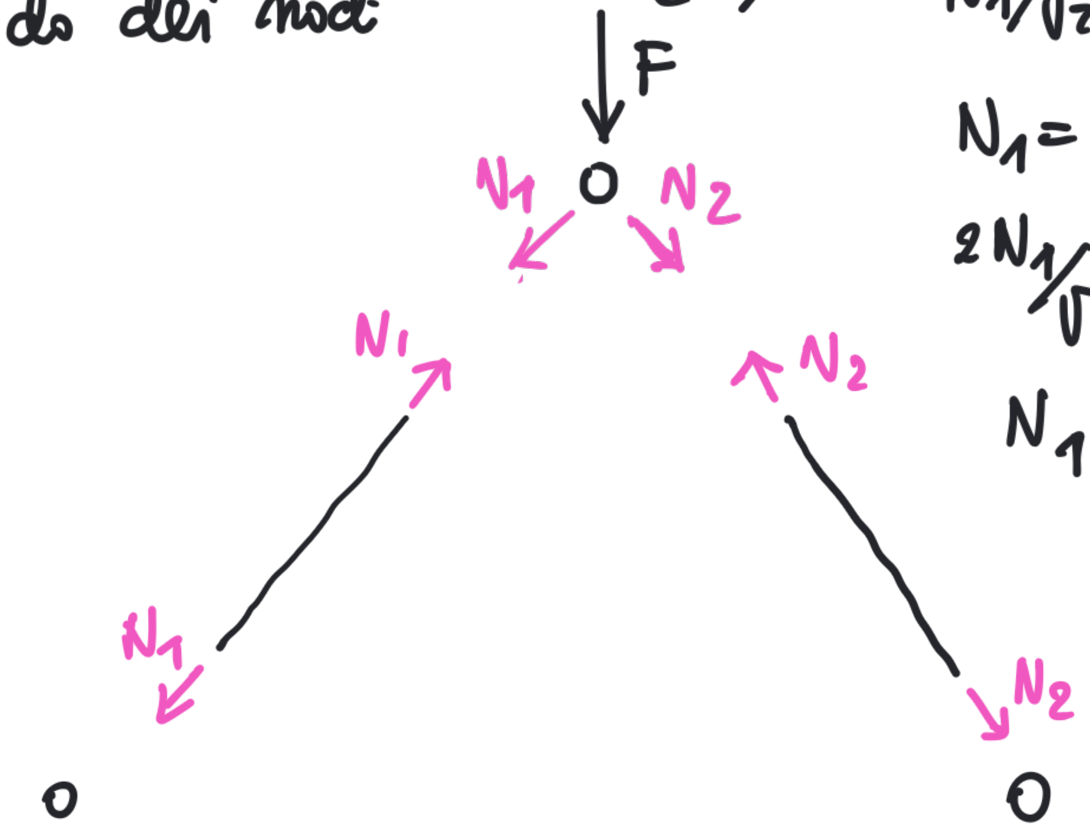
- Motivazione dei criteri di resistenza → esempio di verifica di RESISTENZA

- Criterio di Tresca
- Criterio di Huber-Hencky-von Mises
- Criterio di Galilei

Verifica di resistenza per una travatura reticolare

metodo dei nodi




(→) $-N_1/\sqrt{2} + N_2/\sqrt{2} = 0$

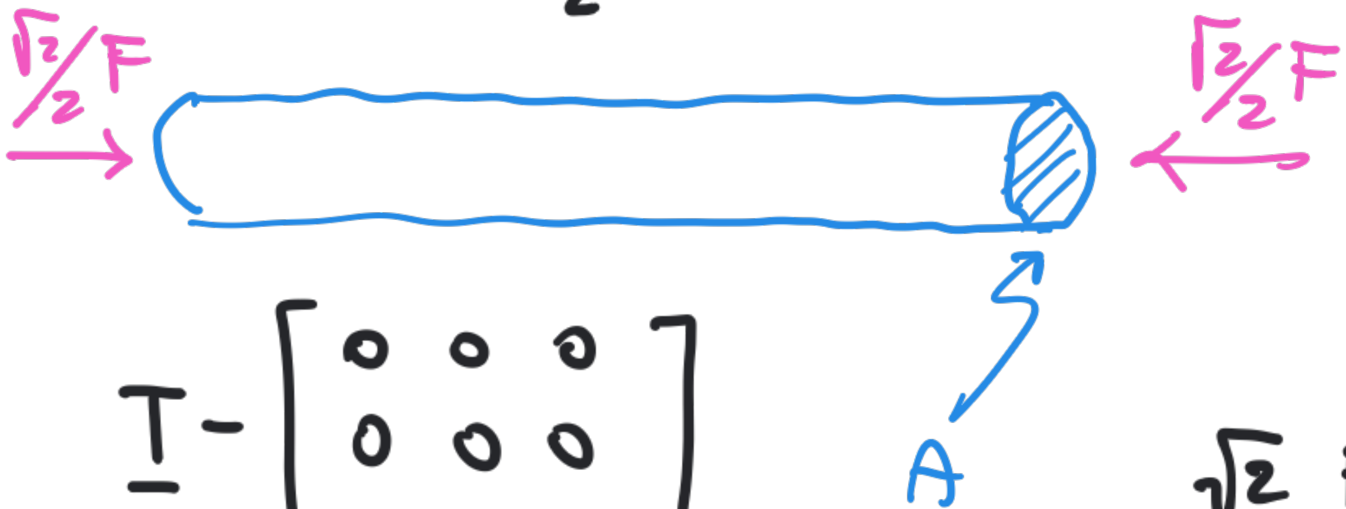
(↑) $-F - N_1/\sqrt{2} - N_2/\sqrt{2} = 0$

$N_1 = N_2$

$2N_1/\sqrt{2} = -F$

$N_1 = -\frac{\sqrt{2}}{2} F$

$$N_1 = N_2 = -\frac{\sqrt{2}}{2} F =: N$$




$$\sigma_z = \frac{N}{A} = -\frac{\sqrt{2}}{2}\frac{F}{A} < 0$$

$$-\sigma_0 \le \sigma_z \le \sigma_0$$

$$\underline{T} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

$$\frac{\sqrt{2}}{2}\frac{F}{A} \le \sigma_0 \Rightarrow \boxed{A > \frac{\sqrt{2}}{2}\frac{F}{\sigma_0}}$$

$\underline{T}$ uniassiale $\Rightarrow$ $\exists$ cambio di coordinate

$$\underline{T} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

verifica: $\underset{\uparrow}{\sigma_C} \leq \sigma \leq \underset{\uparrow}{\sigma_T}$ (*)

tabulati

In sintesi:

la verifica di resistenza per uno stato uniassiale si riduce a controllare che l'unica tensione principale non nulla verifichi le condizioni (*)

$\underline{T}$ uniassiale $\Rightarrow$ $\exists$ cambio di coordinate

$$\underline{T} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

verifica: $\sigma_C \leq \sigma \leq \sigma_T$ (*)

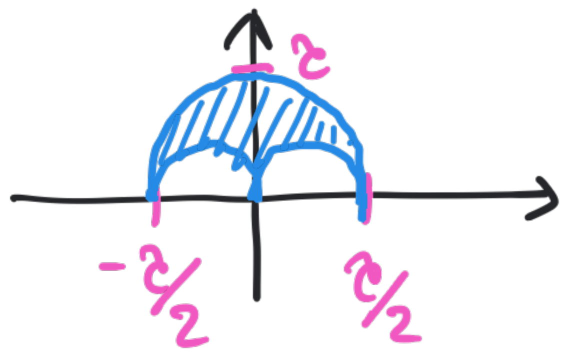
tabulati

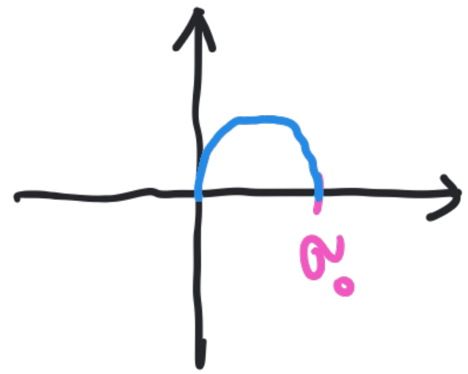
In sintesi:

la verifica di resistenza per uno stato uniassiale si riduce a controllare che l'unica tensione principale non nulla verifichi le condizioni (*)

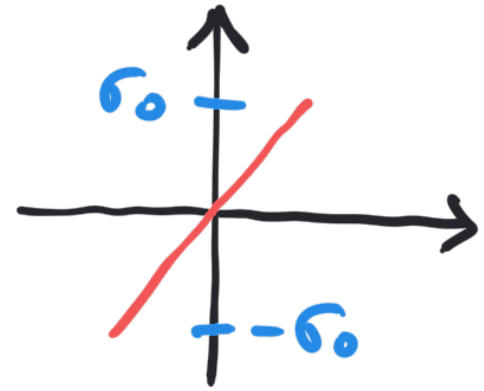
Problema: valutare la severità di uno stato tensionale arbitrario, ad esempio:

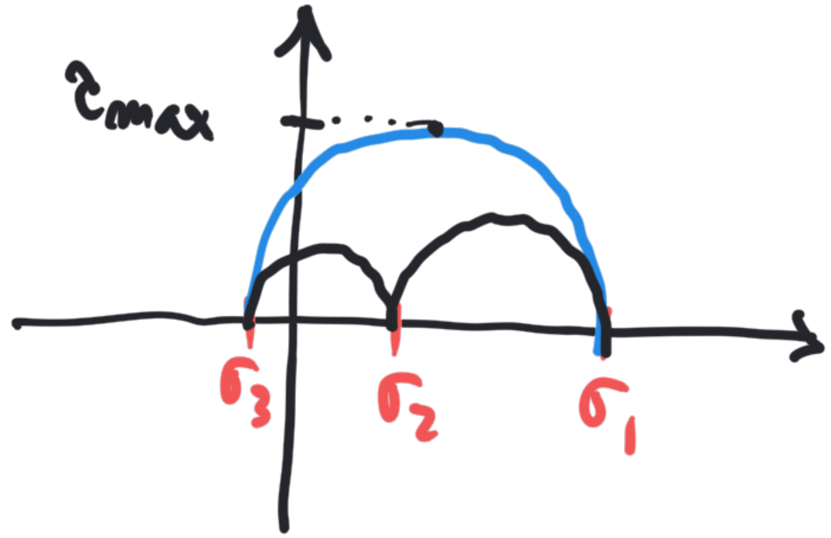
$$\underline{T} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



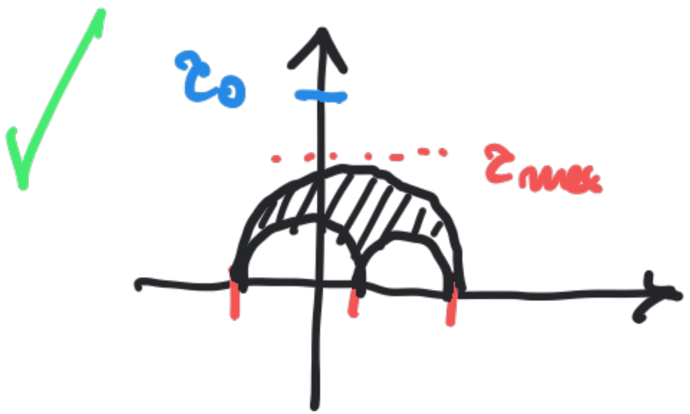
# Criterio di Tresca (materiali duttili)

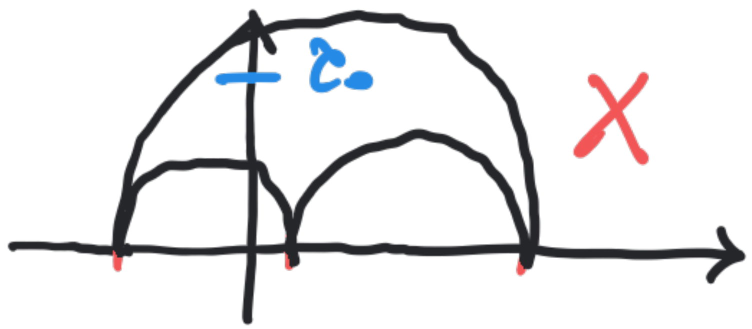


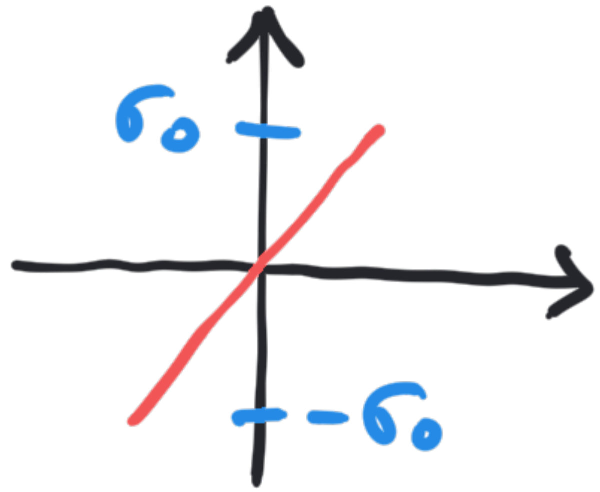

$$\tau_{max} = \max\left\{ \frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_1 - \sigma_3|}{2}, \frac{|\sigma_2 - \sigma_3|}{2} \right\}$$

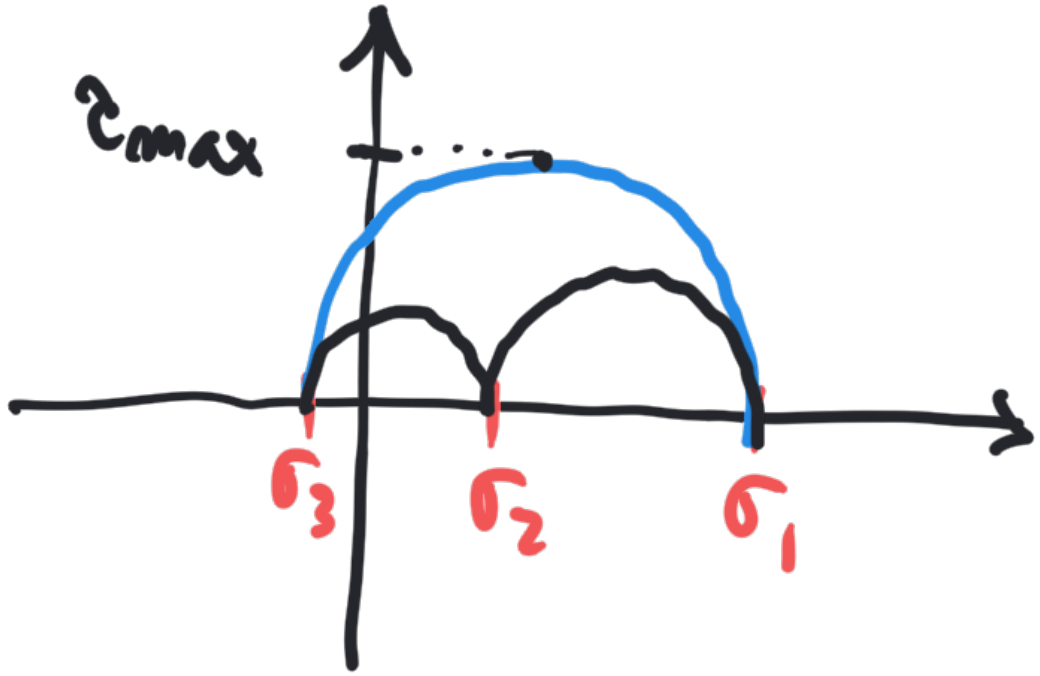
$\tau_{max} \leq \tau_0 \rightarrow$ soglia di snervamento



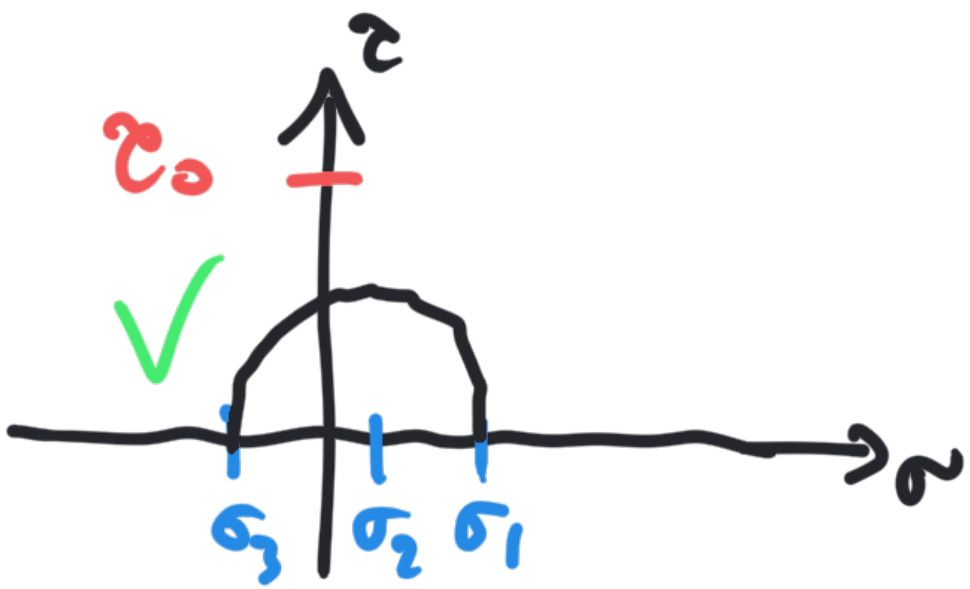
# Criterio di Tresca (materiali duttili)

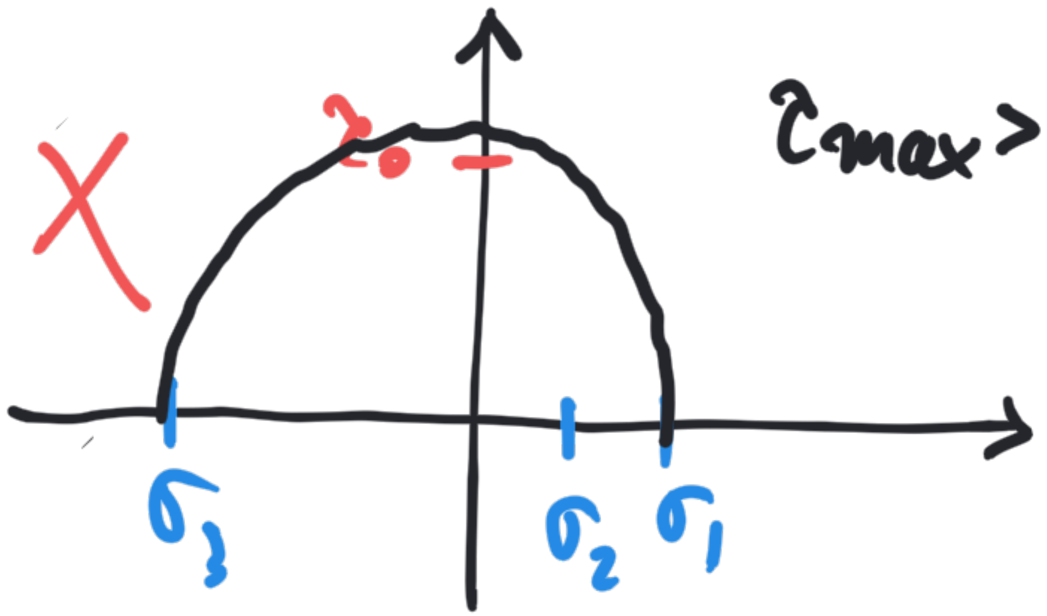



$$\tau_{max} = \max\left\{ \frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_1 - \sigma_3|}{2}, \frac{|\sigma_2 - \sigma_3|}{2} \right\}$$

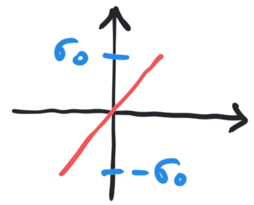
$\tau_{max} \leq \tau_0 \rightarrow$ soglia di snervamento

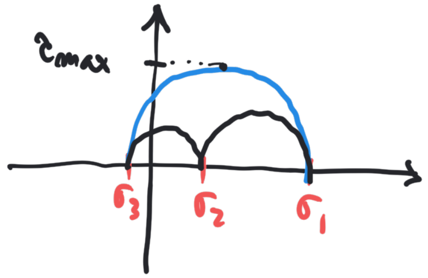



$\tau_{max} > \tau_0$

## Criterio di Tresca (materiali duttili)

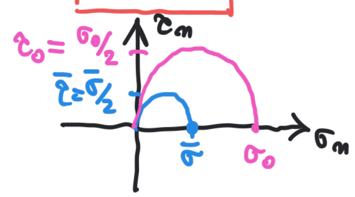


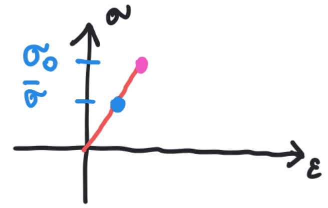




$$\tau_{max} = \max\left\{ \frac{|\sigma_1-\sigma_2|}{2}, \frac{|\sigma_1-\sigma_3|}{2}, \frac{|\sigma_2-\sigma_3|}{2} \right\}$$

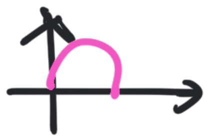
$\tau_{max} \le \tau_0$ → tensione tang. di snervamento.

$$\boxed{\tau_0 = \sigma_0/2}$$



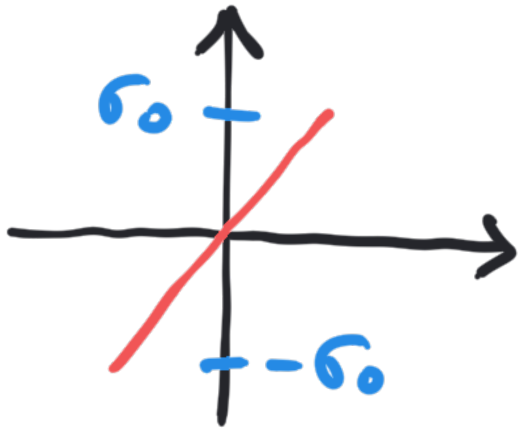



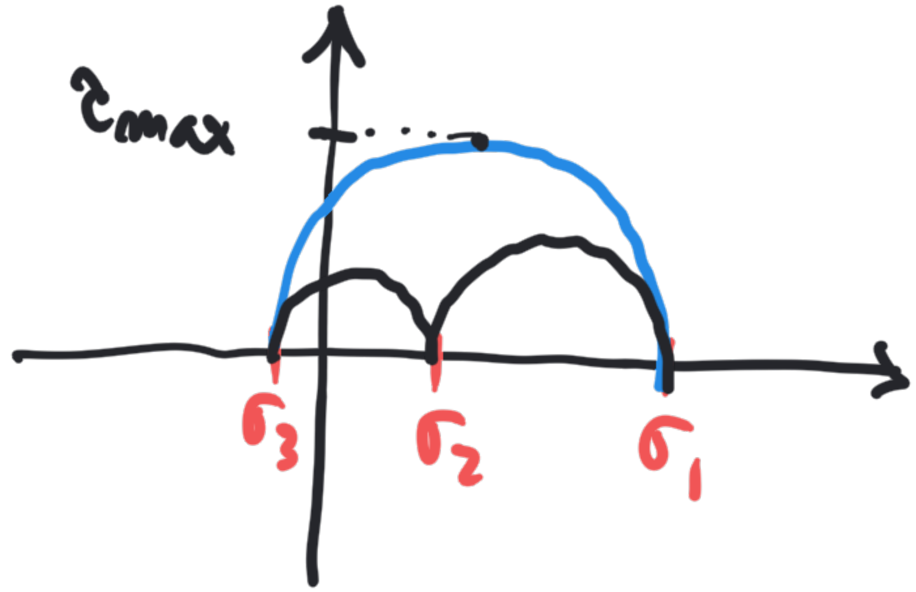

I valori di $\tau_0$ non sono tabulati, ma però si ricavano da $\sigma_0$. Ricordiamo infatti che per uno stato uniassiale del tipo $T = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ la tensione tangenziale massima è $\sigma/2$, come si vede facilmente costruendo l'arbelo di Mohr.



Se $\sigma = \sigma_0$, ovvero se in corrispondenza di tale stato tensionale si raggiunge la soglia di snervamento, la corrispondente tensione tangenziale massima $\sigma_0/2$ deve coincidere con $\tau_0$.

# Criterio di Tresca (materiali duttili)




$$\tau_{max} = \max\left\{ \frac{|\sigma_1-\sigma_2|}{2}, \frac{|\sigma_1-\sigma_3|}{2}, \frac{|\sigma_2-\sigma_3|}{2} \right\} \le \sigma_0/2$$

$\tau_{max} \le \tau_0$ → tensione tang. di snervamento.

$$\boxed{\tau_0 = \sigma_0/2}$$

$$\underline{\underline{T}} \to (\sigma_1, \sigma_2, \sigma_3)$$
$$\downarrow$$
$$\sigma_{idT}$$

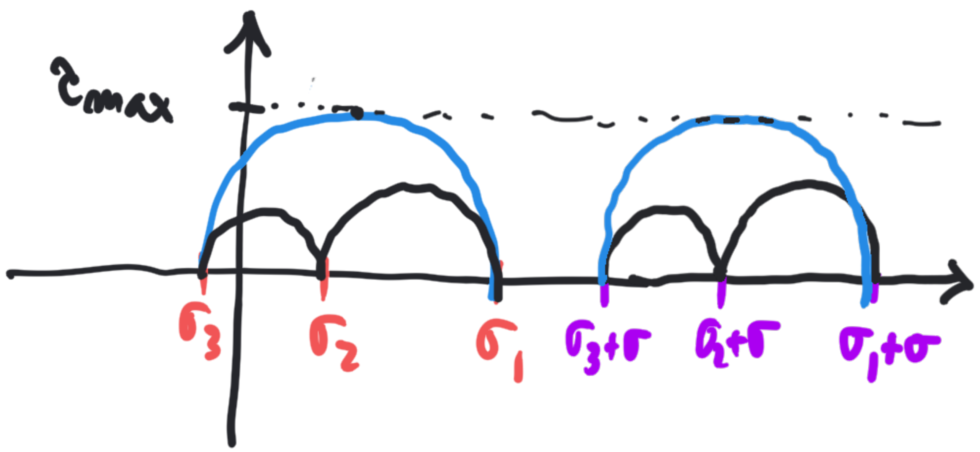
$$\underbrace{\max\{|\sigma_1-\sigma_2|, |\sigma_1-\sigma_3|, |\sigma_2-\sigma_3|\}}_{\sigma_{idT}(\underline{\underline{T}})} \le \sigma_0$$

tensione ideale secondo Tresca

# Criterio di Tresca (materiali duttili)

**OSS 1**:

$$\begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix} = \sigma \underline{\underline{I}}$$




$$\sigma_{idT}(\sigma \underline{\underline{I}}) = 0$$

Stati tensionali idrostatici hanno severità pari allo stato tensionale nullo.

$$\underline{\underline{T}} \mapsto \underline{\underline{T}} + \sigma \underline{\underline{I}}$$

**OSS.2**

La sovrapposizione di uno stato idrostatico non altera la $\sigma_{idT}$, e dunque non cambia la severità di uno stato tensionale.

$$\underbrace{\max\{|\sigma_1-\sigma_2|, |\sigma_1-\sigma_3|, |\sigma_2-\sigma_3|\}}_{\sigma_{idT}(\underline{\underline{T}})} \leq \sigma_0$$

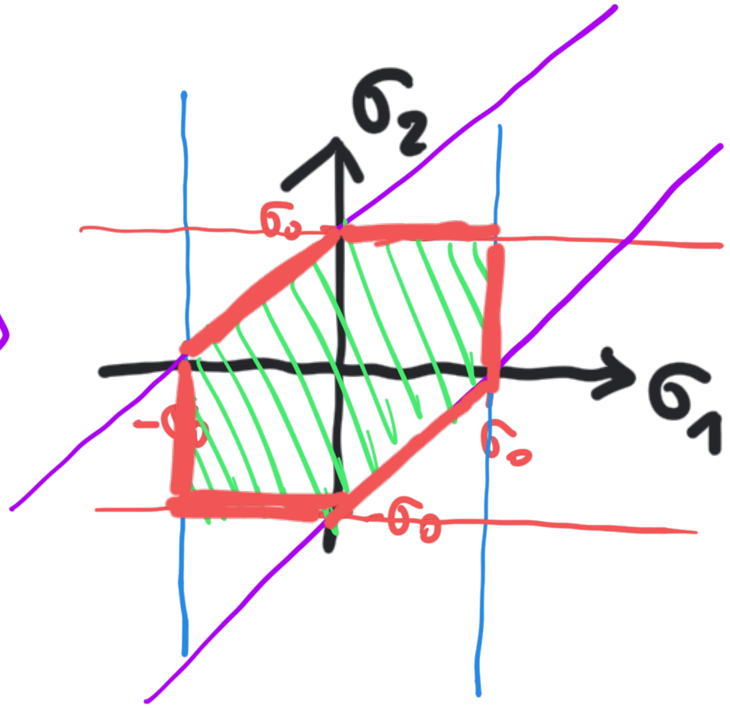
tensione ideale secondo Tresca

$$\boxed{\begin{aligned} -\sigma_0 &\le \sigma_1-\sigma_2 \le \sigma_0 \\ -\sigma_0 &\le \sigma_1-\sigma_3 \le \sigma_0 \\ -\sigma_0 &\le \sigma_2-\sigma_3 \le \sigma_0 \end{aligned}}$$

Caso generale

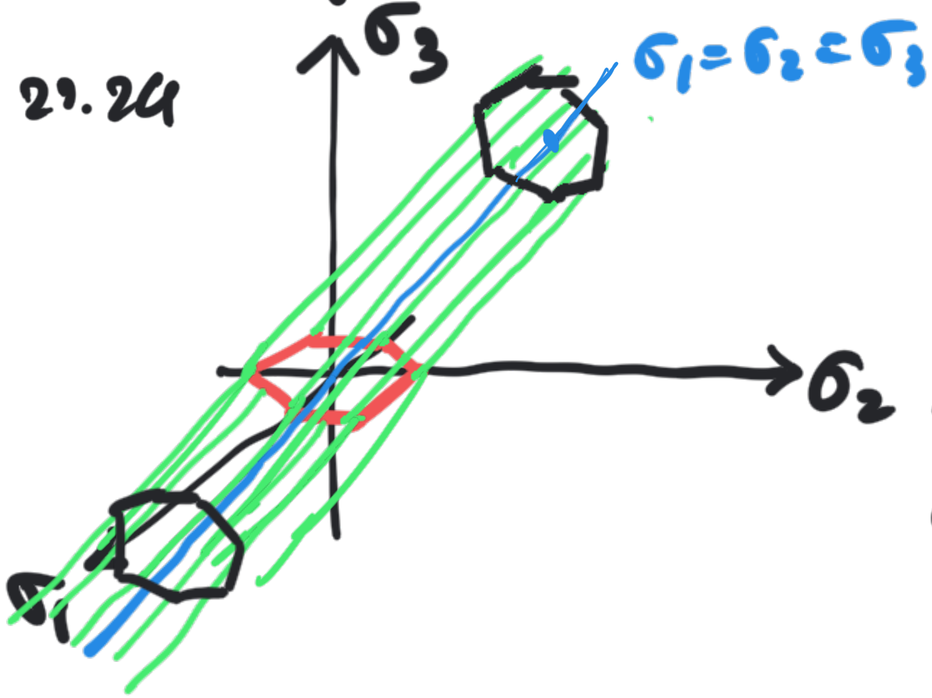
stato piano: $\sigma_3=0$

$$-\sigma_0 \le \sigma_1-\sigma_2 \le \sigma_0$$
$$-\sigma_0 \le \sigma_1 \le \sigma_0$$
$$-\sigma_0 \le \sigma_2 \le \sigma_0$$




esagono di Tresca

Fig 21.24




**OSS.2**

La sovrapposizione di uno stato idrostatico non altera la $\sigma_{idT}$, e dunque non cambia la severità di uno stato tensionale.

$$\underbrace{\max\{|\sigma_1-\sigma_2|, |\sigma_1-\sigma_3|, |\sigma_2-\sigma_3|\}}_{\sigma_{idT}\ (\mathrm{I})} \le \sigma_0$$

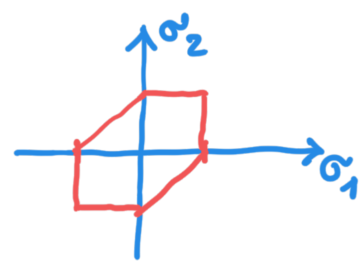
tensione ideale secondo Tresca

$$\begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix} = \sigma \underline{\underline{I}}$$

# Criterio di Huber-Hencky-von Mises (HHM)

$$\tau_{oH} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}$$

$\sigma_3 = 0$




$$\tau_{oH} \leq \tau_{oH}^{0}$$

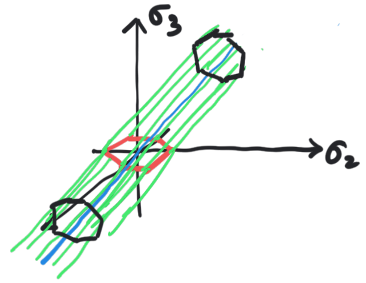
$$\max\{|\sigma_1-\sigma_2|, |\sigma_1-\sigma_3|, |\sigma_2-\sigma_3|\} \leq \sigma_0$$

$$\sigma_{id_T} = \max\{|\sigma_1-\sigma_2|, |\sigma_1-\sigma_3|, |\sigma_2-\sigma_3|\}$$

tensione ideale secondo Tresca

Stato uniassiale : $\sigma_2 = \sigma_3 = 0$

$$\tau_{oH} = \frac{\sqrt{2}}{3}\sigma_1 \Rightarrow \tau_{oH}^{0} = \frac{\sqrt{2}}{3}\sigma_0$$




$$\underbrace{\frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}}_{\sigma_{id\,HHM}} \leq \sigma_0$$

Criterio di Huber-Hencky-von Mises (HHM)

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}$$

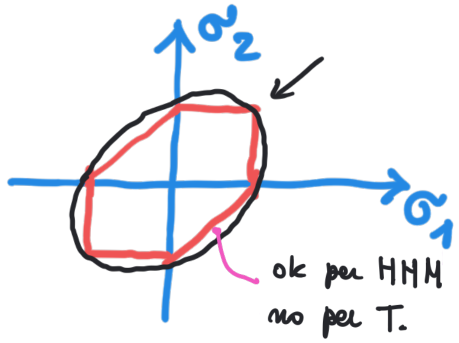
$$\tau_{oct} \le \tau_{oct}^{0}$$

Stato uniassiale : $\sigma_2=\sigma_3=0$

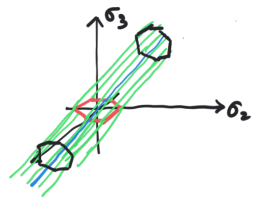
$$\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_1 \Rightarrow \tau_{oct}^{0} = \frac{\sqrt{2}}{3}\sigma_0$$

$$\underbrace{\frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}}_{\sigma_{idHHM}} \le \sigma_0$$

$\sigma_3 = 0$




ok per HHM
no per T.




$$\underbrace{\max\{|\sigma_1-\sigma_2|, |\sigma_1-\sigma_3|, |\sigma_2-\sigma_3|\}}_{\sigma_{idT}} \le \sigma_0$$

tensione ideale secondo Tresca

$\boxed{\text{Tresca più conservativo.}}$

# Criterio di Huber-Hencky-von Mises (HHM)

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}$$

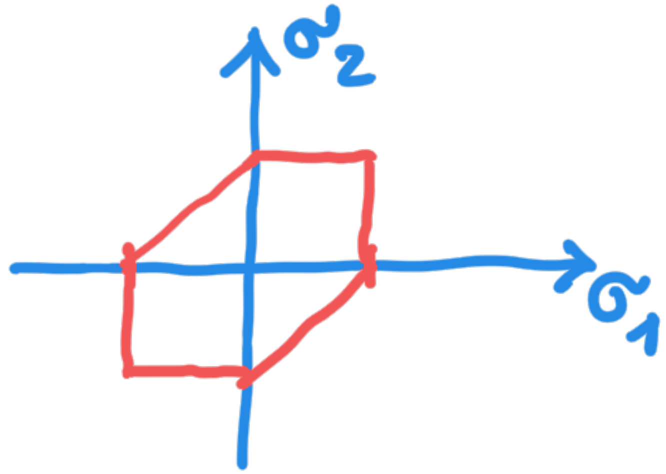
$$\tau_{oct} \leq \tau^0_{oct}$$

Stato uniassiale : $\sigma_2 = \sigma_3 = 0$

$$\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_1 \Rightarrow \tau^0_{oct} = \frac{\sqrt{2}}{3}\sigma_0$$

$$\underbrace{\frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}}_{\sigma_{idHHM}} \leq \sigma_0$$

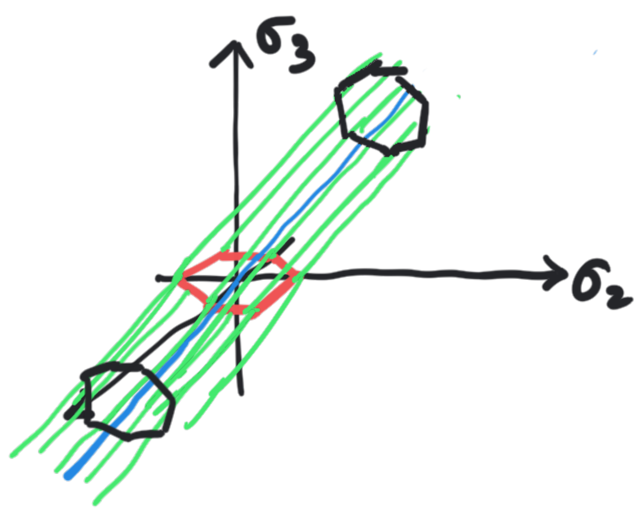
$\sigma_3 = 0$




$$\max\{|\sigma_1-\sigma_2|, |\sigma_1-\sigma_3|, |\sigma_2-\sigma_3|\} \leq \sigma_0$$

$\parallel$

$\sigma_{idT}$ tensione ideale secondo Tresca



# Criterio di Huber-Hencky-von Mises (HHM)

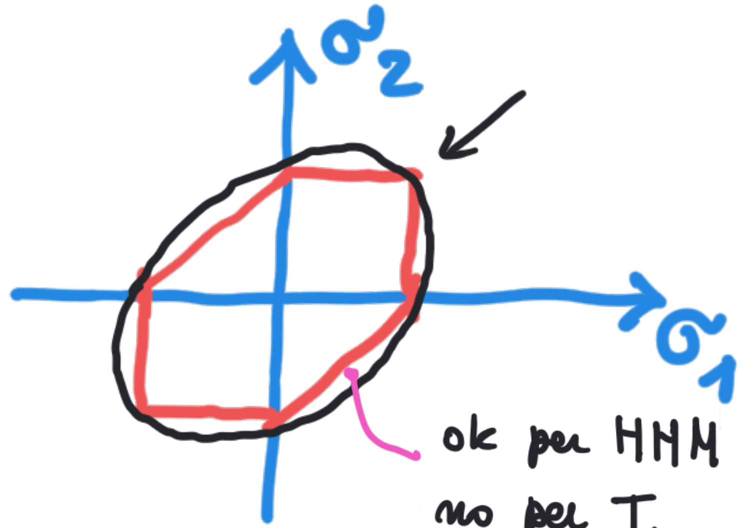
(HHM)

stato piano : $\sigma_3 = 0$

$$\sigma_{idHHM} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$$

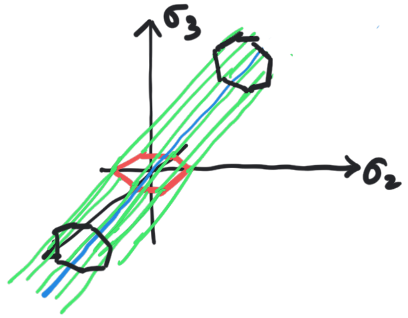
$$\boxed{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \le \sigma_0^2}$$

$\sigma_3 = 0$




ok per HHM
no per T.

$$\underbrace{\frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}}_{\sigma_{idHHM}} \le \sigma_0$$



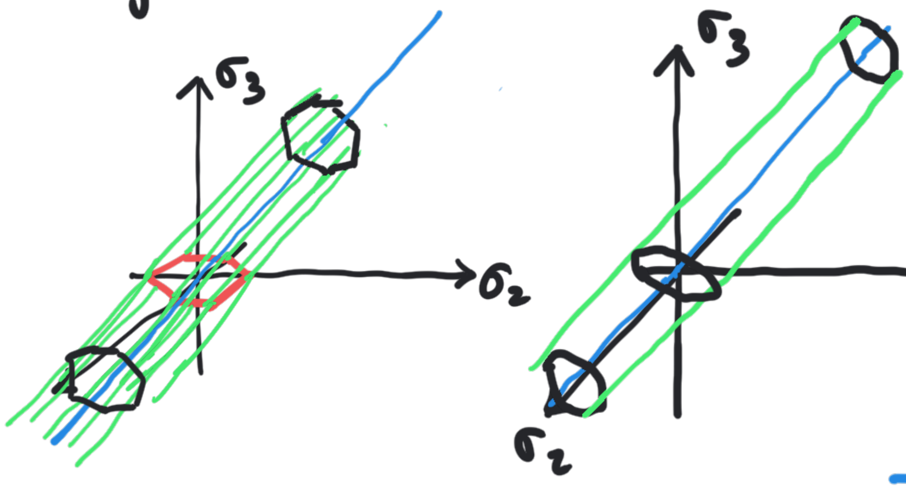

$$\max\{|\sigma_1-\sigma_2|, |\sigma_1-\sigma_3|, |\sigma_2-\sigma_3|\} \le \sigma_0$$

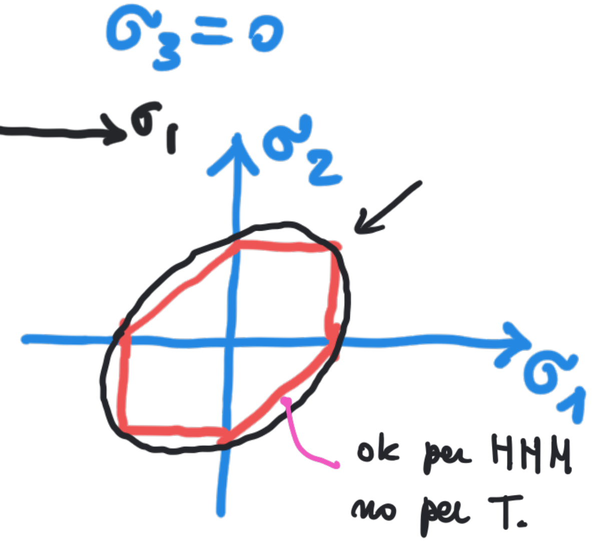
!!

$\sigma_{idT}$ : tensione ideale secondo Tresca

$\boxed{\text{Tresca più conservativo.}}$

Fig. 22.4



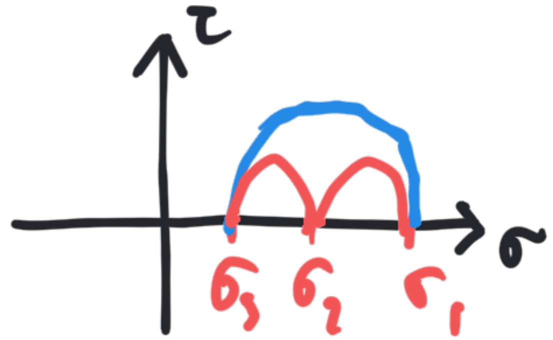

stato piano: $\sigma_3 = 0$

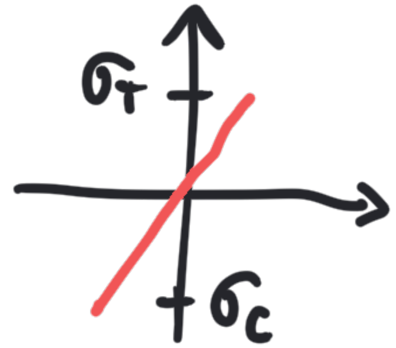
$$\sigma_{id\,HMH} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$$

$$\boxed{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \leq \sigma_0^2}$$

Tresca più conservativo.

Nel caso generale, le tensioni ammissibili secondo HMH descrivono un cilindro avente come asse l'asse idrostatico. Tale cilindro è circoscritto al prisma a base esagonale che descrive gli stati tensionali ammissibili secondo Tresca

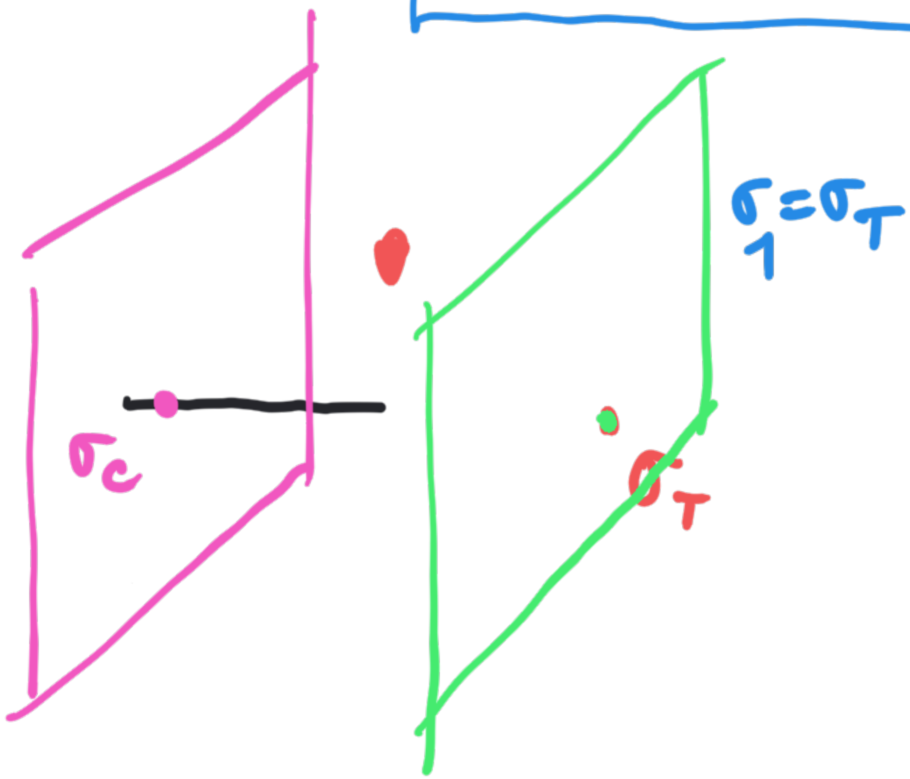


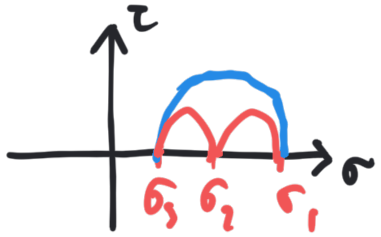

Galileo-Rankine

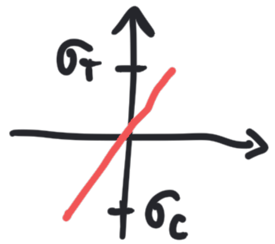
$$\sigma_{min} = \min\{\sigma_1, \sigma_2, \sigma_3\} \qquad \sigma_{max} = \max\{\sigma_1, \sigma_2, \sigma_3\}$$

$$\boxed{\sigma_{min} \geq \sigma_c \qquad \sigma_{max} \leq \sigma_T}$$




$$\sigma_c \leq \sigma_1 \leq \sigma_T$$
$$\sigma_c \leq \sigma_2 \leq \sigma_T \qquad (22.6)$$
$$\sigma_c \leq \sigma_3 \leq \sigma_T$$

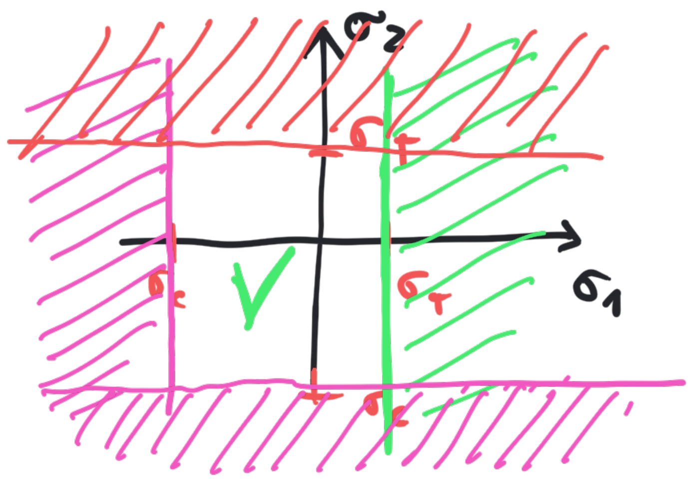



Galileo-Rankine

$$\sigma_{min} = \min\{\sigma_1, \sigma_2, \sigma_3\} \qquad \sigma_{max} = \max\{\sigma_1, \sigma_2, \sigma_3\}$$

$$\boxed{\sigma_{min} \geq \sigma_C \qquad \sigma_{max} \leq \sigma_T}$$




$$\sigma_C \leq \sigma_1 \leq \sigma_T$$

$$\sigma_C \leq \sigma_2 \leq \sigma_T$$

$$\sigma_C \leq \sigma_3 \leq \sigma_T$$

fig 22-3

$\sigma_3 = 0$